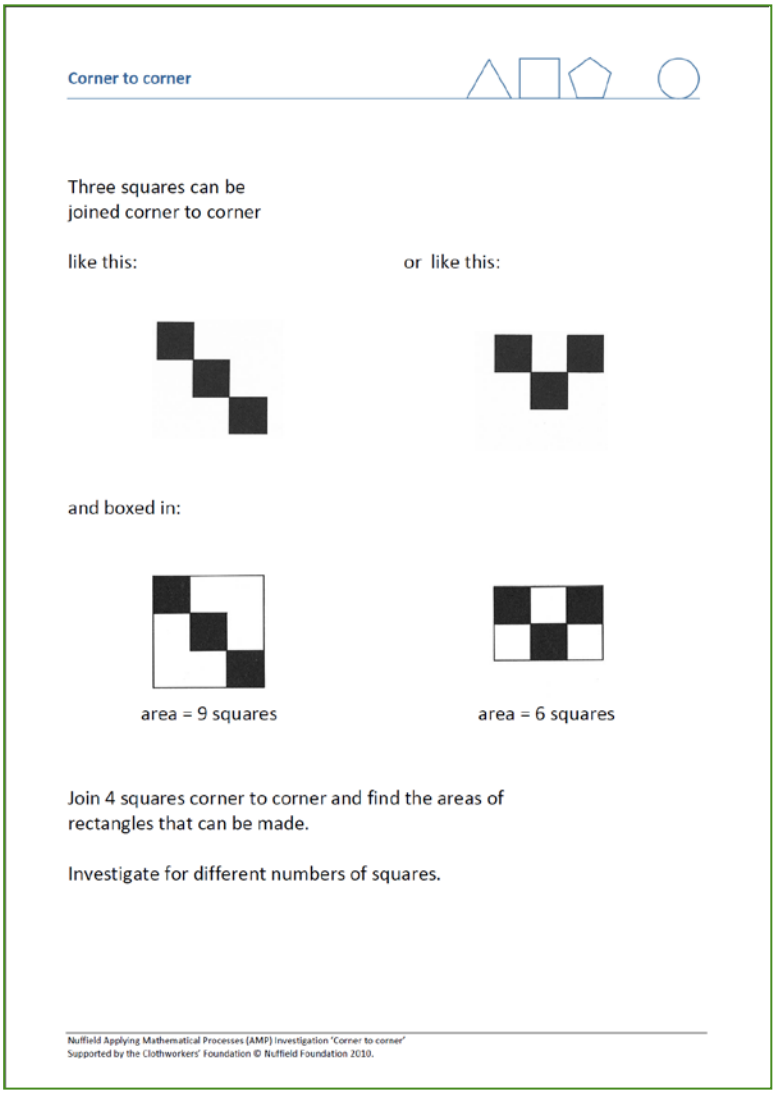
Activity description

Pupils investigate how different numbers of squares can be joined corner to corner, and the effect their arrangement has on the area of the rectangle that encloses the squares.



Corner to corner

Three squares can be joined corner to corner

like this:

or like this:

and boxed in:

area = 9 squares

area = 6 squares

Join 4 squares corner to corner and find the areas of rectangles that can be made.

Investigate for different numbers of squares.

Nuffield Applying Mathematical Processes (AMP) Investigation 'Corner to corner'
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Suitability

Pupils working at all levels; individuals or pairs

Time

1 to 2 hours

AMP resources

Pupil stimulus, Flash and PDF interactives

Equipment

Square dotty paper or squared paper or computer drawing software
Square tiles
Tracing paper or mirrors

Key mathematical language

Rectangle, square, area, congruent, pattern, variable, maximum, minimum, conjecture, justification

Key processes

Representing Determining which aspects to investigate and record; using diagrammatic and other representations to further the investigation.

Analysing Working systematically, making accurate mathematical diagrams, identifying patterns and beginning to make generalisations.

Interpreting and evaluating Considering the findings to form convincing arguments and justifying generalisations.

Communicating Explaining the approach taken and the outcomes at each stage of the work.

Teacher guidance

The provided Flash interactive can be used to introduce the task dynamically. Have pupils arrange 3 squares in different ways, showing the box being drawn around them. Alternatively use the Pupil Stimulus sheet.

Explain that squares/tiles must be joined corner to corner and *not* edge to edge, identifying through observation and discussion that, when squares are correctly joined, their edges must have a right angle between them.

Ensure that pupils are able to differentiate the 'boxed in' shapes as rectangles and squares, as well as appreciate the fact that squares are a subset of rectangles.

Pupils may wish to decide for themselves whether an m x n rectangle and an n x m rectangle are to be treated as being equivalent, but emphasise the need to declare their 'ruling' and the need to adopt it consistently.

Pupils can use computer drawing software, the provided PDF interactive, or square tiles to make the activity more practical and to record their work.

The investigation need not be simply to find minimal and maximal area configurations but to explore the range of configurations and related patterns.

Probing questions and feedback

AMP activities are well suited to formative assessment, enabling pupils to discuss their understanding and decide how to move forward. See www.nuffieldfoundation.org/whyAMP for related reading.

- Encourage pupils to draw together their various observations and conjectures and to present them clearly and coherently. Stress the importance of explaining their choices and their reasoning.
- What are you trying to find out?
- Have you found all the different ways? How can you be certain?
- What are the maximum or minimum areas for each number of tiles?
- You've made a prediction. How will you decide if you are correct?
- For a given number of tiles, is there a configuration for every (whole number) area between the minimum and maximum areas? Is there more than one configuration for a given area?
- Can you find any general rules? Can you explain why your rules are true?

Extensions

- Consider the number of possible ways of joining squares within the constraints of the task – counting and classifying the various configurations for a fixed number of squares joined corner to corner.
- Work with shapes other than squares. What are the similarities and differences in results obtained?
- How could you change the rule to create a new meaningful activity?

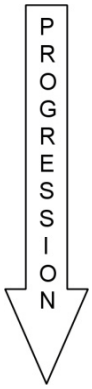
Progression table

The table below can be used for:

- sharing with pupils the aims of their work
- self- and peer-assessment
- helping pupils review their work and improve on it.

The table supports formative assessment but does not provide a procedure for summative assessment. It also does not address the rich overlap between the processes, nor the interplay of processes and activity-specific content. Please edit the table as necessary.

Representing *Making choices about what to investigate and how to record*	**Analysing** *Working systematically and accurately, identifying patterns*	**Interpreting and evaluating** *I Making conjectures, generalising and providing justifications*	**Communicating and reflecting** *Quality of discussion of both methods and outcomes*
Represents the activity using diagrams with squares correctly joined and boxed in Pupil A	Finds all possible configurations for a given number of squares	Makes simple observations about their findings Pupil B	Labels diagrams so that someone else can understand the results Pupil A
Considers variation across a limited range of examples Pupils B, C	Identifies simple numerical patterns Pupil B	Makes valid and relevant observations, based on a range of results Pupils C, D	Brings together findings effectively, e.g. uses a table of results or gives a simple and relevant conclusion Pupils B, C
Chooses to investigate properties for any number of joined squares Pupil D	Explores more than one property when varying the number of squares Pupil C	Explains occurrence of simple patterns	Expresses a valid general finding, providing fair evidence using diagrams, words or symbols Pupil D
Chooses to use a systematic approach and structured sets of diagrams to explore finer aspects	Organises their investigation so that several different factors can be explored efficiently Pupil D	Justifies a general rule, e.g. relating to maximum or minimum area	Describes clearly what was investigated and explains how conclusions were arrived at



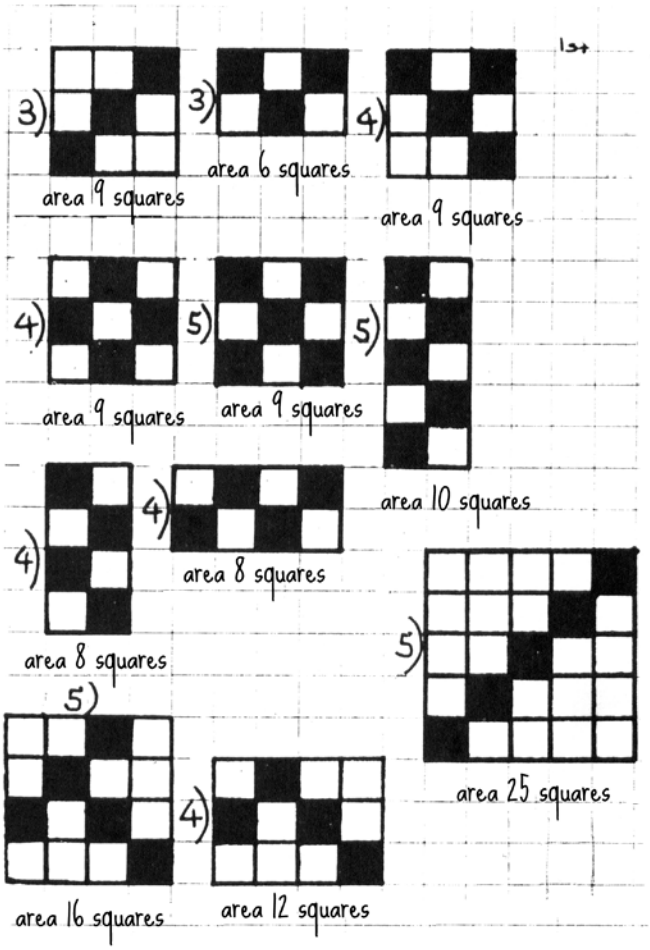
Sample responses

Pupil A

Pupil A has drawn 4 of the 5 possible arrangements of 4 squares joined corner to corner (modulo rotations and reflections). Rectangles are drawn round the outside and the areas correctly identified.

She has started to investigate 5 joined squares.

She has not looked for relationships or made observations on the results.

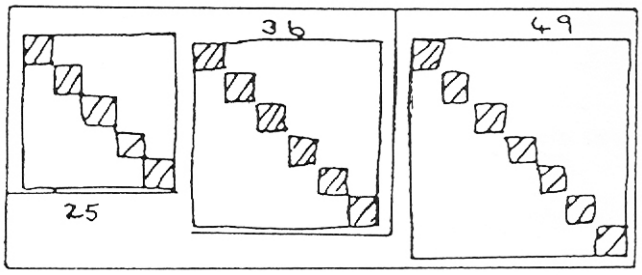


Probing questions

- Have you found all arrangements for 4 squares? How can you be sure?
- What approach have you used to organise your investigation?
- How can you best summarise what you have observed so far?

Pupil B

Pupil B chooses to investigate squares joined diagonally, recognising that this represents the maximum area possible for a given number of joined squares. Findings are summarised effectively, but more justification of the observed relationship is needed.



Relationship between No. of squares and the maximum of the area is that you just square the number of squares for the maximum area.

eg

$1^2 = 1$
$2^2 = 4$
$3^2 = 9$
$4^2 = 16$
$5^2 = 25$

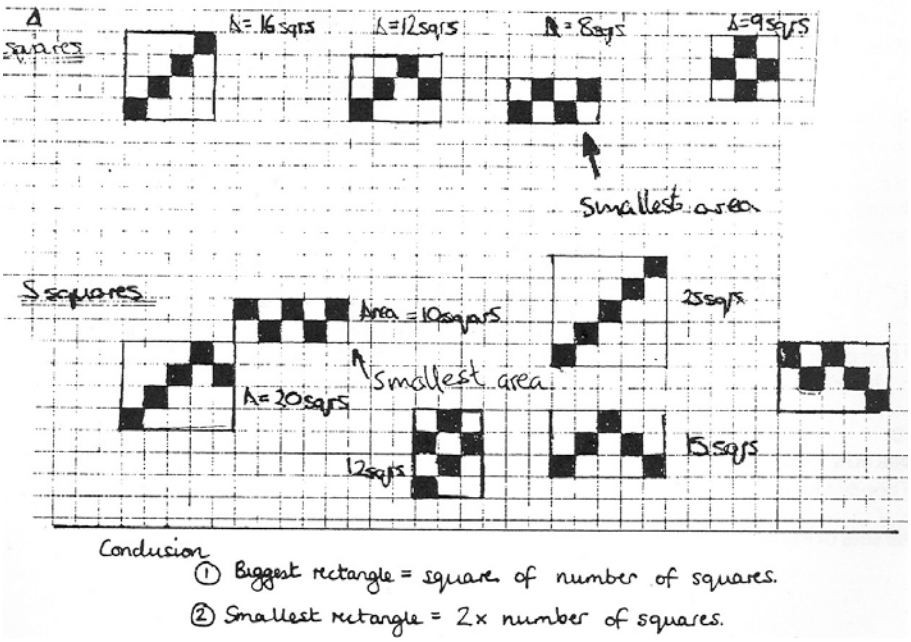
Probing questions

- Explain why your arrangement gives the maximum area.
- What other properties could you investigate?

Pupil C

Pupil C seeks relationships, focusing on maximum and minimum areas. Not all cases are presented and the approach is unclear.

The pupil develops conjectures, terming them conclusions, though only based on two sets of observations.



Probing questions and feedback

- Pupil C would benefit from a discussion of conjectures, verification and justification. Considering more cases, using a systematic approach, could help identify general principles that could help establish their observations.
- How did you go about gathering configurations? How can you guarantee that you have found all cases for a given number of squares?
- What observations led you to your conclusions? Will they be true for further numbers of squares? How can you establish that?

Pupil D

Pupil D has systematically followed a particular line of enquiry. A set of accurate results is presented concisely in a table.

General rules are communicated, though there is no explanation of how the results or conclusions were generated, nor are there any justifications for rules or exceptions.

	Possible ways	Surrounding rectangles	MIN AREA	MAX AREA
1	1	1	1	1
2	1	1	4	4
3	2	2	6	9
4	5	4	8	16
5	12	7	9	25

The rule for maximum area is n^2

The rule for minimum area is usually $2n$. When there are exceptions (eg 1, 5, 8) the rule is $2n-1$.

Probing questions

- How did you reach your conclusions?
- Can you craft a more general rule for minimum area that accounts for the exceptions?
- Why do your rules work?