Activity description

This Decision Mathematics activity includes two problems based on the Chinese postman problem.

The College Open Day problem provides an introduction to the concept, and asks students to investigate the minimum distance someone would have to travel to deliver leaflets along all the streets near to a college, starting at and returning to College.

The second problem, called Easter Parade, requires students to find an Eulerian trail for a network with four odd nodes.

Suitability

Level 3 (Advanced)

Time

1-2 hours

Resources

Student information sheet, worksheet
Optional: slideshow

Equipment

Calculators
Optional: internet access

Key mathematical language

Algorithm, graphs, nodes, vertices, arcs, edges, traversable, Eulerian trail, semi-Eulerian, network

Notes on the activity

This activity can be used to introduce or revise the Chinese postman algorithm.

The slideshow includes the same examples and background information.

The student sheets and slides can be used to introduce this topic, as follows.

- First use Slides 1 to 3 and page 1 of the student sheets to introduce the context. Ask students to try to find a route that starts from the college and travels along each edge exactly once. Some students may be able to find one of the possible routes ending at G.
- Use slides 4 to 7 to give background information and definitions (or omit these if you wish).
- Slide 8 returns to the original problem. At this stage you could ask

students to complete pages 2 and 3 of the student sheets. Students who have already found one semi-Eulerian trail from C to G could try to find another route.

- Use slides 8 and 9 to aid class discussion and give the answers to the College Open Day.
- Use slide 10 to summarise the Chinese postman algorithm – there is also a summary on page 3 of the student sheets.
- Use slide 11 to introduce the Easter Parade activity. Ask students to try page 4 of the student sheets. Continue slide 11 followed by slide 12 to show one of the possible solutions to this problem.
- Slide 13 has the Reflection questions.

During the activity

Students can be asked to work on the problems in pairs and to discuss their findings, for example, the different semi-Eulerian trails for the College Open day activity and Eulerian trails for the Easter parade.

Points for discussion

Why all the vertices have to be even in order for the graph to be Eulerian.

Why the sum of the vertices on any network will be even.

Why, in any network, the number of odd vertices must be even.

You may also like to include that:

- when finding a route, the number of times each vertex is visited is (order ÷ 2), except for the start/finish which is visited (order ÷ 2) + 1.
- the number of ways of pairing n odd vertices is $1 \times 3 \times 5 \times 7 \times \ldots \times (n - 1)$
- as the number of odd nodes increases, the number of possible pairings to be considered increases dramatically. It is therefore impractical to do the algorithm by hand with more than 6 nodes. Even with a computer, it can become too time consuming – with 12 odd nodes there are 10395 combinations to consider.

Extensions

Students can be asked to check whether their Eulerian trail conforms to the rule for the number of times each vertex should occur (given above).

Use a Search engine to find other Chinese postman problems on the internet.

Answers

Answers to the two problems are given in the slideshow.