Drug clearance

Activity description

This activity explores the rate of drug clearance from the body, both in respect of common analgesic compounds and of caffeine. It shows that if we assume the rate at which the drug is cleared from the body is proportional to the amount of drug present in the body, then the relationship between the drug level and time is exponential.

Suitability and Time

Level 3 (Advanced)

1–2 hours

Resources

Student information sheet, worksheet
Optional slideshow

Equipment

Calculator, graph paper or a graphic calculator or computer spreadsheet

Key mathematical language

Rate, differential equation, maximum, laws of logarithms, half-life, exponential

Notes on the activity

The information sheet shows how the level of paracetamol decays in a body after an initial dose.

The equation $\dfrac{\mathrm{d}m}{\mathrm{d}t} = -km$

is used as a starting point, leading to $\ln m = -kt + c$.

Using the initial conditions m = initial dose when t = 0, the value for c is found.

The half-life of the drug is then used to calculate k and thus find equations for both t and m.

http://sonet.nottingham.ac.uk/rlos/bioproc/halflife/index.html

provides a good series of animations to explain half-life, though not quite in the same way as on the information sheet.

During the activity

Ensure that students know why each step follows from the preceding one.

Check that students remember the rules of logarithms, and ask them why it is better to leave $\ln 500$ in this form, rather than calculate its value.

Points for discussion

At the point when half-life is mentioned, it would be useful to explore the half-life of various common analgesics – see the notes in Extensions below. You could draw students' attention to the table in **Try these** no 2, showing the wide variation in impact of caffeine on people with different characteristics. You could suggest that other drugs may also vary in their impact between people.

In addition, the half-life does not imply that the drug is becoming inactive at this point. Ibuprofen has one of the shortest half-lives, but is still effective after 6 – 8 hours.

Discuss the graph of m against t and how this would vary with different doses and half-lives.

Ask if students can suggest the shape of the graph if someone takes repeat doses every 6 hours.

Extensions

Note that the half-life of paracetamol is about 2 hours regardless of whether it is taken in soluble or solid forms.

For aspirin and its active products (the active chemicals it breaks down into), the time varies with the dose. For example 300 mg has a half-life of 2–3 hours, but higher doses can have as high a half-life as 15 hours or more.

Information on this and other drugs can be found on the internet, for example at

http://www.rxmed.com/b.main/b2.pharmaceutical/b2.1.monographs/CPS-%20Monographs/CPS-%20%28General%20Monographs-%20M%29/MOTRIN%20IB.html

The half-life of ibuprofen is about 2 hours, but it is clinically effective for between 6 and 8 hours

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provides quite a good series of animations to explain half-life, but not in quite the same way as it is explained on the information sheet.

Answers

1a $\dfrac{\mathrm{d}m}{\mathrm{d}t}$ is the rate of change of the drug level.
It is equal to $-km$ because the drug level is falling and the rate of fall is proportional to the amount of drug remaining, m.

ci First drug 278 minutes, second drug 119 minutes

ii First drug 125 mg, second drug 32.7 mg

1d i See the graph on the right.

1d ii

Both graphs show exponential decay. The intercepts are 500 mg and 300 mg, the peak values of the drugs.

The gradient of both graphs are much steeper at the beginning than later, indicating that the drug is cleared at a slower and slower rate as time goes by.

Drug clearance
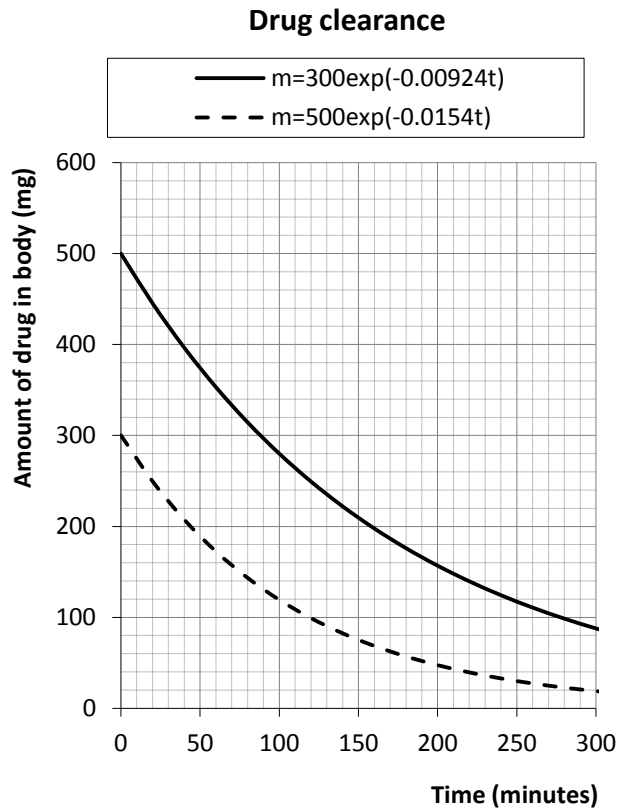
2 The models for each person and each drink are as shown in the table.

Person	Drink	Model
Non-smoking adult	Tea	$70e^{-0.116t}$
	Filter Coffee	$150e^{-0.116t}$
	Instant Coffee	$90e^{-0.116t}$
	Cola	$35e^{-0.116t}$
Smoker	Tea	$70e^{-0.173t}$
	Filter Coffee	$150e^{-0.173t}$
	Instant Coffee	$90e^{-0.173t}$
	Cola	$35e^{-0.173t}$
Woman using oral contraceptive pill	Tea	$70e^{-0.0578t}$
	Filter Coffee	$150e^{-0.0578t}$
	Instant Coffee	$90e^{-0.0578t}$
	Cola	$35e^{-0.0578t}$
Pregnant woman	Tea	$70e^{-0.0385t}$
	Filter Coffee	$150e^{-0.0385t}$
	Instant Coffee	$90e^{-0.0385t}$
	Cola	$35e^{-0.0385t}$
Child	Tea	$70e^{-0.231t}$
	Filter Coffee	$150e^{-0.231t}$
	Instant Coffee	$90e^{-0.231t}$
	Cola	$35e^{-0.231t}$