How frequent are earthquakes?

Earthquakes can be devastating, sometimes causing huge loss of life and destruction. How frequent are earthquakes? It isn't straightforward to answer this question as there are thousands of very minor earthquakes occurring every day. But there are far fewer high magnitude earthquakes.

In this activity you will use functions and graphs to model a relationship between the magnitude of earthquakes and how frequently they occur.

Information sheet

Charles Richter developed the Richter Scale for measuring the magnitude of earthquakes in 1935. The table below shows the average annual frequency of earthquakes for a given magnitude (based on data from 1900 to 1990).

Description	Magnitude	Average Annual Frequency
Great Earthquakes	8 and higher	1
Major Earthquakes	7 – 7.9	18
Strong Earthquakes	6 – 6.9	120
Moderate Earthquakes	5 – 5.9	800
Light Earthquakes	4 – 4.9	6 200 (estimated)
Minor Earthquakes	3 – 3.9	49 000 (estimated)
Very Minor Earthquakes	2 – 2.9	approx 1 000 per day
	1 – 1.9	approx 8 000 per day

Data Source:
US National Earthquake Information Center

In order to model this data we need to identify a set of data points.

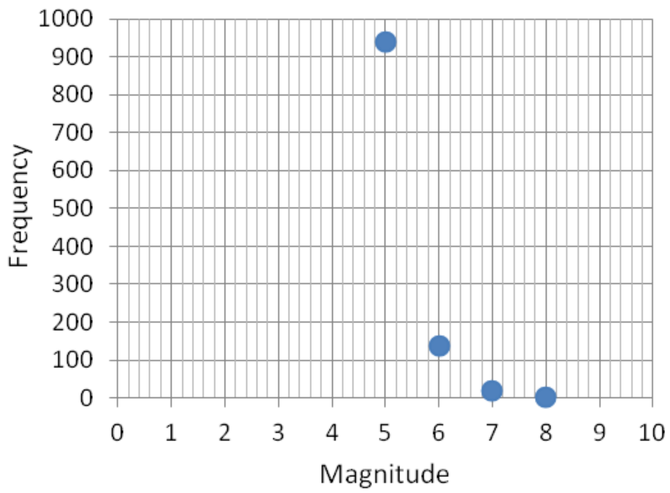
We will define *M* to be magnitude and *N* to be the average number of earthquakes per year of magnitude at least *M*. The graph shows the first few data points from the table.

Think about

How can we convert magnitudes from a range to a single value for a data point?

Average annual frequency (*N*) of earthquakes of magnitude at least *M*

M	*N*
8	1
7	19
6	139
5	939
4	7 100*
3	56 000
2	421 000
1	3 341 000



* The values for *N* have been rounded to the same degree of accuracy as the original data.

Think about…

What does this graph tell you about the relationship between *M* and *N*?

Can you suggest some functions that you could use to model the data?

A. Exponential relationships

If we don't know what function might be used to model data, one approach is to look at differences or ratios of successive values. The table below shows the ratio of frequencies for successive magnitudes.

M	8	7	6	5	4	3	2	1
N	1	19	139	939	7100	56000	421000	3341000
ratio	0.05	0.14	0.15	0.13	0.13	0.13	0.13	

[1 ÷ 19]

With the exception of the first value, these ratios are all of the same order of magnitude, and they are close in value. This suggests that the relationship between *M* and *N* is exponential and we could model the data using a function of the form $N = ka^M$, with some suitable k and with $a \approx 0.13$.

Think about

Why might the first value be significantly different from the other ratios?

Taking logs of both sides of the equation $N = ka^M$ gives

$$\log N = \log ka^M$$

And using the laws for manipulating logarithms,

$$\log N = \log k + M \log a$$

Think about...

If $\log N$ is plotted against M, what shape will the graph be?

What will the graph tell us about $\log k$ and $\log a$?

B. Base of logarithm

For any positive base b, except for b = 1, and any number x>0, we can find $\log_b x$, as this is the power that b must be raised to in order to get x. For example,

$\log_2 8 = 3, \log_{10} \frac{1}{100} = \ -2, \text{and} \ \ \log_9 3 = \frac{1}{2}.$

The bases used most commonly are *e* and 10. However, many scientific calculators can work with a given valid base.

One advantage of base 10 is that we can read off the order of magnitude of the original number. For example, if $\log_{10} x$ is between 1 and 2 then we know that x is between 10 and 100, and similarly, if x is between 1000 and 10 000, then we know $\log_{10} x$ is between 3 and 4. This is useful for estimating and checking calculations.

Worksheet

Try these

1. Calculate $\log_{10} N$ for each of the values of N and complete this table:

M	1	2	3	4	5	6	7	8
N	3341000	421000	56000	7100	939	139	19	1
$\log_{10} N$								

2. Plot a graph of $\log_{10} N$ against M for the full set of data.
Your scale should be as large as possible.

a Draw the line of best fit.

b What is the value of the intercept on the $\log_{10} N$ axis?

c What is the gradient of the line?

Remember your answers should not be given to a greater degree of precision than the original data.

3. Write down the equation of your line.

a Use this equation to write N as a power of 10.

Hint: $\log_{10} x$ is the inverse of the function 10^x.

b Now use the laws for manipulating powers to write N in the form

$N = ka^M$.

c Check how well the relationship you have just found fits the data.

4. Use the data in the original table to find a relationship between the mid-points of the magnitude intervals and the average annual frequency.

Reflect on your work

How did plotting the log graph help you to model the relationship between magnitude and frequency?

If plotting a log graph from a set of data points gives a straight line, does that mean the data can be modelled by an exponential function?

Could you have used $\ln N$ instead of $\log_{10} N$ to help you find a relationship between M and N? Would it have affected the model?