## Large and small



Sometimes it is necessary to work with very large numbers and very small numbers. Here are two examples.

### Jupiter's surface area

Jupiter's surface area is equal to  $4\pi r^2$ where Jupiter's radius r = 71492000 metres.

#### Think about...

• how to work out Jupiter's surface area.

### **Atomic particles**

This is 1836 times as heavy as an electron.

#### Think about...

- how to work out the mass of an electron
- how to work out the number of protons needed to make a mass of 1 kg.

This activity shows how to use standard form to work out the answers to questions like these.

## **Information sheet**

### A Powers of 10

10 <sup>3</sup>	10 <sup>2</sup>	10 <sup>1</sup>	10 <sup>0</sup>	10 <sup>1</sup>	10 <sup>-2</sup>	10 <sup>-3</sup>
$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$
1000	100	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$

Other examples are  $10^6 = 1\ 000\ 000$  and  $10^{-9} = \frac{1}{1\ 000\ 000\ 000}$ 

Think of a negative in the power as 'one over'.

#### Think about...

Any number to the power zero has a value of 1, so  $1\ 000\ 000^0 = 1$  and  $4^0 = 1$ Can you explain why this is so?



Image of Jupiter produced by STScI for NASA from http://hubblesite.org/



Image of an oxygen atom produced by the NASA/Goddard Space Flight Center Scientific Visualization Studio from <u>http://www.nasa.gov</u>

# **B** Calculating with standard form

Numbers written in standard form have two parts  $10^{n}$  where n is between 1 and 10 and n is a positive or possible.

 $a \times 10^{n}$ , where a is between 1 and 10 and n is a positive or negative integer

Examples:  $7.3 \times 10^5 = 730\ 000$  or  $3.681 \times 10^{-7} = 0.000\ 000\ 3681$ 

Calculators have a variety of ways of entering numbers in standard form:

Using EXP key:	For $4.5 \times 10^3$ enter 4.5 EXP 3 and for $7.1 \times 10^{-5}$ enter 7.1 EXP ( – ) 5
Using EE key:	For $4.5 \times 10^3$ enter $4.5 \text{ EE} 3$ and for $7.1 \times 10^{-5}$ enter $7.1 \text{ EE} (-) 5$
Using 10 <sup>x</sup> key:	For $4.5 \times 10^3$ enter $4.5 10^x$ 3 and for $7.1 \times 10^{-5}$ enter $7.1 10^x$ (-) 5

Note: Not all calculators have a separate (-) key. If your calculator does not have this, enter (-5) using brackets.

Use your calculator to work out the following examples, and check that you get the same answers:

- 1  $(4 \times 10^{-3}) + (8 \times 10^{-5}) = 4.08 \times 10^{-3} \text{ or } 0.00408$
- **2**  $(7 \times 10^5) (9 \times 10^4) = 6.1 \times 10^5$  or 610 000
- **3**  $(4 \times 10^3) \times (2 \times 10^6) = 8 \times 10^9$  or 8 000 000 000
- 4  $(8 \times 10^9) \div (2 \times 10^6) = 4 \times 10^3 \text{ or } 4000$

# C Using standard form in real situations

### Jupiter's surface area

Jupiter's radius r = 71492000 metres. In standard form this is  $7.1492 \times 10^7$ . The surface area is  $4\pi r^2 = 4\pi \times (7.1492 \times 10^7)^2$ Work this out on your calculator. You should get  $6.4228 \times 10^{16} \text{ m}^2$ Think about... What is this as an ordinary number?

### **Atomic particles**

The mass of a proton is 0.000 000 000 000 000 000 000 001 673 g. In standard form this is  $1.673\times10^{-24}$ 

This is 1836 times as heavy as an electron.

### Think about...

how to work out the mass of an electron

If you find this difficult, think of an easier problem:

A father weighs 84 kg. This is 3 times as heavy as his daughter. How would you work out how much his daughter weighed?

The mass of an electron =  $(1.673 \times 10^{-24}) \div 1836$ 

Work this out on your calculator. You should get  $9.112\times10^{-28}\,g$  What is this as an ordinary number?

#### Think about...

• how to work out the number of protons needed to make 1 kg

If you find this difficult, think of an easier problem.

Satsumas weigh on average 40 g each. How do you work out how many would weigh 1 kg?

The number of protons that weigh 1 kg =  $1000 \div (1.673 \times 10^{-24})$ 

Work this out on your calculator. You should get  $5.977 \times 10^{\,26}$  What is this as an ordinary number?

# **Try these**

- **1** Write these in standard form:
- a three million b £12.6 million c eighty thousand
- d five billion e £3.25 billion f ten and a half billion
- g seven million, five hundred and twenty thousand
- h four hundred and fifty million and sixty-eight thousand
- 2 Write these as ordinary numbers.

а	$2.4  imes 10^6$	<b>b</b> $2.4 \times 10^{-6}$	$\textbf{c} \ 3\times 10^9$
d	$3  imes 10^{-9}$	<b>e</b> $7.14 \times 10^{5}$	<b>f</b> $7.14 \times 10^{-5}$

3 Write these numbers in standard form.

а	36 000 000	<b>b</b> 482 000	<b>c</b> 900 000 000
d	0.000 25	e 0.000 007	<b>f</b> 0.003 456

# Think about...

In the following problems, if you are not sure what calculation you need, make up a situation with easy numbers to help you to see what to do.

4 At the end of 2009 there were  $6.4 \times 10^8$  mobile broadband subscriptions and  $4.9 \times 10^8$  fixed broadband subscriptions in the world. What was the total number of broadband subscriptions?

5 The Earth is  $1.5 \times 10^{11}$  metres from the Sun. Light travels at about  $3 \times 10^8$  metres per second.

Use the formula:  $time = \frac{distance}{speed}$  to estimate the time taken for sunlight to reach the Earth.

6 The wavelength of red light is  $7 \times 10^{-7}$  metres and that of violet light is  $4 \times 10^{-7}$  metres. What is the difference between these wavelengths?



7 A bacterium measures  $1 \times 10^{-6}$  metres and a virus measures  $2 \times 10^{-7}$  metres. How many times bigger is the bacterium than the virus?

8 A flea is  $9.5 \times 10^{-4}$  metres long and a dust mite is  $2.3 \times 10^{-3}$  metres long. Which of these is longer and by how much?

9 The surface area of the Earth is approximately  $5.1 \times 10^8$  square kilometres.

The area of Europe is about  $9.9 \times 10^6$  square kilometres. Approximately what % of the Earth's area is Europe?

10 Each locust in a swarm eats about  $2.3 \times 10^{-3}$  kilograms of food in a day and there are about  $3 \times 10^4$  locusts in the swarm.

Roughly how much does the whole swarm eat in a day?

**11** You may know that Saturn has rings. It also has moons. The table gives information about six of Saturn's moons.

Moon	Distance from Saturn (m)	Volume (m <sup>3</sup> )	Mass (kg)
Dione	$3.77 \times 10^{8}$	$7.36 \times 10^{17}$	$1.05 \times 10^{21}$
Enceladus	$2.38 \times 10^{8}$	$6.54 \times 10^{16}$	$8.40 \times 10^{19}$
lapetus	$3.56 \times 10^9$	$1.63 \times 10^{18}$	$1.88 \times 10^{21}$
Rhea	$5.27 \times 10^{8}$	$1.88 \times 10^{18}$	$2.49 \times 10^{21}$
Tethys	$2.95 \times 10^{8}$	$6.24 \times 10^{17}$	$7.55 \times 10^{20}$
Titan	$1.22 \times 10^{9}$	$7.15 \times 10^{19}$	$1.35 \times 10^{23}$



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Image from <a href="http://www.nasa.gov">http://www.nasa.gov</a>
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a List the moons in the order of their distance from Saturn. Start with the nearest.

- **b** List the moons in the order of their volumes. Start with the largest moon.
- c List these moons in the order of their masses. Start with the heaviest.
- **d** Use the formula:  $density = \frac{mass}{volume}$  to find the density of each moon.

**12** The table gives the atomic masses of carbon, hydrogen, oxygen and nitrogen.

a List the elements in the order of their masses. Start with the lightest.

**b** Molecules of other substances are made from combinations of atoms. Find the mass of each of the following molecules:

Element
Atomic Mass (kg)

Carbon
 $1.99 \times 10^{-26}$  

Hydrogen
 $1.67 \times 10^{-27}$  

Oxygen
 $2.66 \times 10^{-26}$  

Nitrogen
 $2.33 \times 10^{-26}$ 

i carbon monoxide (CO) made from one atom of carbon plus one of oxygen

ii carbon dioxide (CO<sub>2</sub>) made from one atom of carbon plus two of oxygen

iii water ( $H_2O$ ) made from two atoms of hydrogen plus one of oxygen

- **c** i Which is heavier, a molecule of acetic acid  $(C_2H_4O_2)$  or nitric acid  $(HNO_3)$ ?
- ii What is the difference in their masses?

**13** It has been said that "Unravel your DNA and it would stretch from here to the Moon". (<u>http://hypertextbook.com/facts/1998/StevenChen.shtml</u>). Check whether this is true.

You can assume that:

- there are about  $10^{13}$  cells in the body; each cell contains about 3 billion base pairs, where each base pair is  $3.4 \times 10^{-10}$  metres long.
- the distance between the Earth and the Moon is  $3.8 \times 10^8$  metres.

# At the end of the activity

Can you explain why:

a  $2.42 \times 10^4 = 24\ 200$ b  $2.42 \times 10^{-4} = 0.000242.$ c  $0.000\ 056 = 5.6 \times 10^{-5}$ d  $1\ 250\ 000\ 000 = 1.25 \times 10^9$ 

How do you enter a number like  $5.6 \times 10^{-5}$  into your calculator? What are the advantages of working in standard form? Are there any disadvantages?