Activity description

This activity can be used to introduce the shape and main features of proportional, linear, inverse proportional, and quadratic graphs

Suitability

Level 2 (Higher)

Could also be used at the beginning of a Level 3 (Advanced) courses.

Time

Up to 3 hours for the full activity.
However, you may prefer to split it into two parts:
linear graphs and non-linear graphs.

Resources and equipment

Student information sheet, worksheet, spreadsheet

Computer access with Excel available

Key mathematical language

Equation, graph, constant, spreadsheet, proportional, linear,
inverse proportional, quadratic

Notes on the activity

For the full activity, each student will need to have a copy of all the student sheets and access to Excel. The Excel file is protected, the password being fsma.

If you decide to split the activity into two parts:

Linear graphs
Students need the information and activity sheets A to D (pages 1 and 2)

Non-linear graphs
Students need the information sheet and activity sheets A, E and F
(pages 1 and 3–5)

During the activity

If you have an activeboard or equipment to project the spreadsheet onto a screen, you could start by showing students how to use the scroll bars to change the values of the constants in the equations. On the linear and quadratic graphs, clicking on an arrow at the end of a scroll bar increases or reduces the constant by 0.1, whilst clicking on other parts of the bar increases or reduces the constant by 1.

On the inverse proportional graph, clicking on an arrow at the end of the scroll bar increases or reduces the constant by 0.5, whilst clicking on other parts of the bar increases or reduces the constant by 5.

Alternatively a constant can be varied by dragging the central bar along the scroll bar.

Projecting the spreadsheet will also aid class discussion.

This could include looking at patterns in the values of x and y in the tables, as well as the main features of the graphs.

Students should be aware that, in many real contexts, the value of the variables is often restricted, usually to positive values. This means that plotting data may only give part of a curve.

Answers/Points for discussion

1 Lines with equations of the form $y = mx$ always pass through (0, 0) that is the origin.

2 As m increases from 0 to 5, the gradient of the line increases, that is the line becomes steeper.

3 The gradient now becomes negative, with the line getting steeper as m decreases from 0 to –5.

4 Lines with equations of the form $y = c$ have zero gradient.
As c increases from – 4 to 4, the line moves up the page from $y = -4$ to $y = 4$.

5 For lines with equations of the form $y = x + c$, as c increases from –4 to 4, the point of intersection of the line with the y axis moves from $y = -4$ to $y = 4$ (and the gradient is 1).

6 For lines with equations of the form $y = -x + c$, as c increases from –4 to 4 the point of intersection of the line with the y axis moves from $y = -4$ to $y = 4$ (and the gradient is –1).

Reflection

For all lines with equations of the form $y = mx + c$,
c gives the intercept on the y axis.

For all lines with equations of the form $y = mx + c$, m gives the gradient.

The question "Can you think of a real life situation that might be modelled by an equation of the form $y = mx + c$?" may only be appropriate if students have previously plotted graphs from real situations. If so, discuss examples such as the time for roasting a joint (20 minutes + 20 minutes per pound), or a plumber's charge (call-out charge + price per hour).

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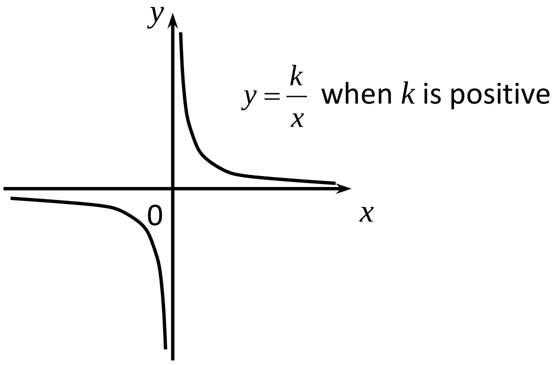

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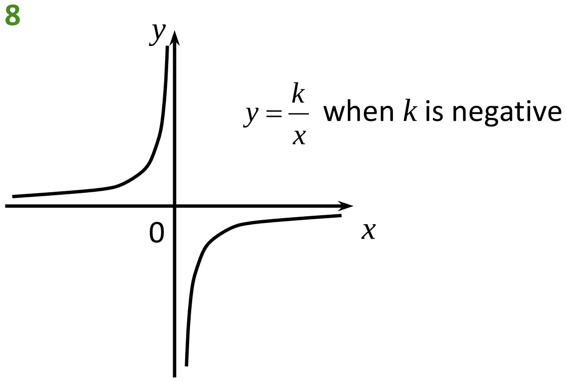

9 The curve never passes through (0, 0).

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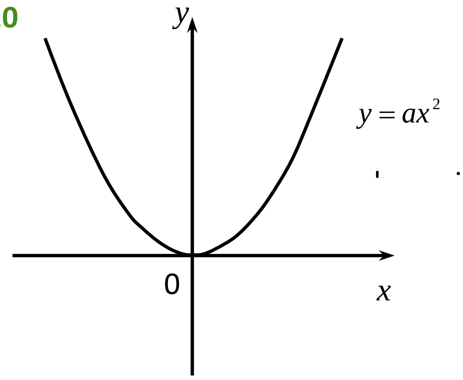

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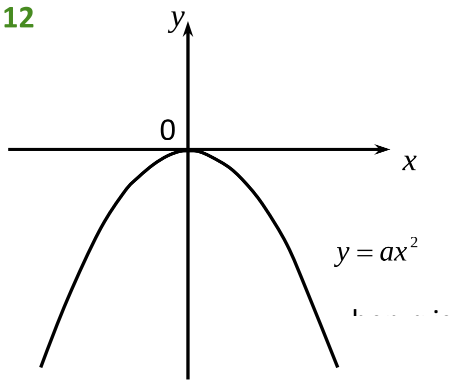

11 As a increases from 0 to 5, the curve 'narrows' and gets steeper.

13 As a decreases from 0 to –5, the curve 'narrows' and gets steeper.

14 As c increases from –4 to 4, the point of intersection of the curve with the y axis moves from –4 to 4.

15 As c increases from –4 to 4, the point of intersection of the curve with the y axis moves from –4 to 4.

16 For quadratic curves with equations of the form $y = ax^2 + c$, c gives the intercept on the y axis.

The value of a affects the gradient and orientation of the curve.

17 As b increases from 0 to 4, the curve moves to the left and downwards.

In fact quadratic curves with equations of the form $y = x^2 + bx$ always cross the x axis at the points (0, 0) and ($-b$, 0). The minimum point is $\left(-\frac{b}{2}, -\frac{b^2}{4}\right)$.

18 As b decreases from 0 to –4, the curve moves to the right and downwards.

19 As b increases from 0 to 4, the curve moves to the right and upwards.

In fact quadratic curves with equations of the form $y = -x^2 + bx$ always cross the x axis at the points (0, 0) and (b, 0). The maximum point is $\left(\frac{b}{2}, \frac{b^2}{4}\right)$.

20 As b decreases from 0 to –4, the curve moves to the left and upwards.

Reflection

Findings could include:

- The intercept on the y axis is always equal to c.
- In general, changing either a or b changes the gradient as well as the position of the turning point of the curve.
- In the special case when a = 0, the graph is a straight line (unless b and c are also 0).
- When a is positive, increasing b always moves the turning point of the curve to the left.
- When a is negative, increasing b always moves the turning point of the curve to the right.

Notes The turning point of a quadratic curve is given by $\left(-\frac{b}{2a}, c-\frac{b^2}{4a}\right)$.

The gradient of a quadratic curve is given by $2ax + b$.