Activity description

Pupils investigate the placing and number of fire hydrants required in a city with square blocks that form a rectangular grid.

Suitability

Pupils working at all levels; individuals or pairs

Time

1 to 2 hours

AMP resources

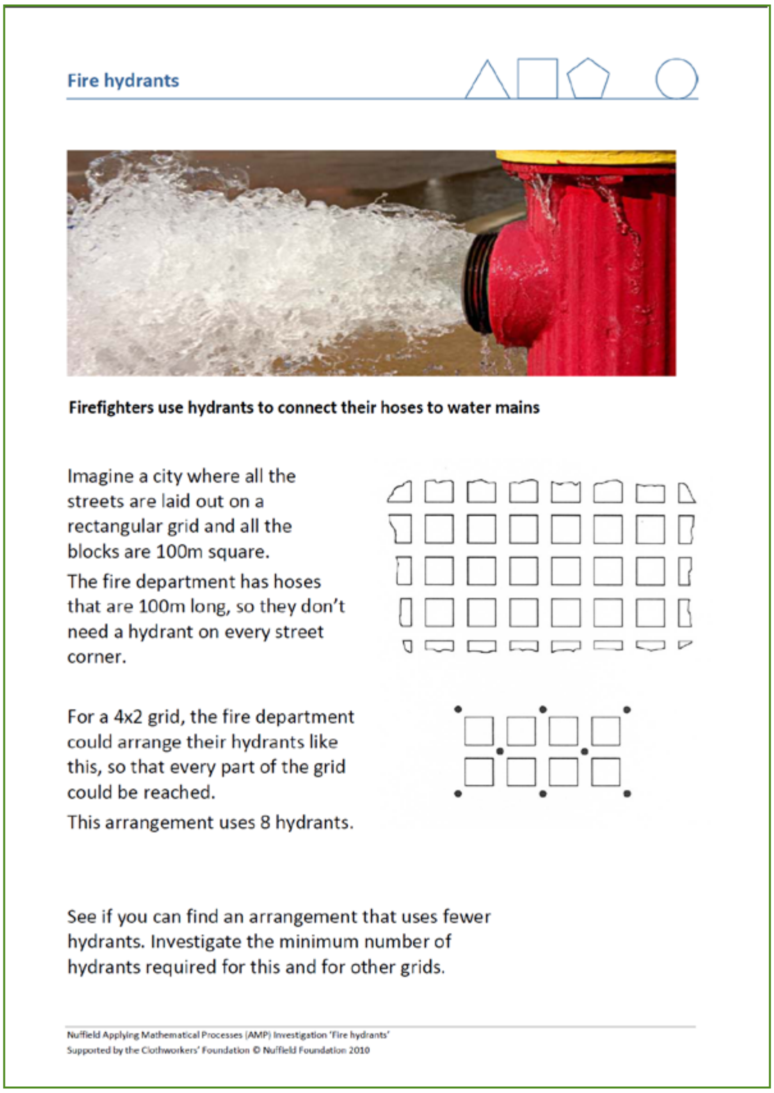
Pupil stimulus
Slideshow
Flash and PDF interactive

Equipment

Square grid (or dotty) paper
Mini-whiteboards
Counters or multilink cubes or square tiles
Straws or pipe cleaners

Key mathematical language

Minimum, grid, arrangement, odd, even, symmetry, variable, exception



Fire hydrants

Firefighters use hydrants to connect their hoses to water mains

Imagine a city where all the streets are laid out on a rectangular grid and all the blocks are 100m square.

The fire department has hoses that are 100m long, so they don't need a hydrant on every street corner.

For a 4x2 grid, the fire department could arrange their hydrants like this, so that every part of the grid could be reached.

This arrangement uses 8 hydrants.

See if you can find an arrangement that uses fewer hydrants. Investigate the minimum number of hydrants required for this and for other grids.

Nuffield Applying Mathematical Processes (AMP) Investigation 'Fire hydrants'
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Key processes

Representing Diagrammatic representation, and moving to more abstract mathematical methodology.

Analysing Considering different arrangements; making accurate mathematical diagrams; working systematically; identifying patterns; beginning to make generalisations.

Interpreting and evaluating Considering findings to form convincing arguments, and relating these to the context of the task and diagrams / patterns found; justifying findings.

Communicating and reflecting Explaining the approach taken and outcomes achieved at each stage of the work.

Teacher guidance

The activity can be introduced using the slideshow, and demonstrated interactively using the flash interactive. One could also use pupils as fire hydrants, with a square grid on the floor. If necessary, explain to pupils what a fire hydrant is.

Use the map of downtown Windermere, Florida, as stimulus for discussion, or consider parts of New York or Milton Keynes.

Set the context for the investigation. A city built to a grid plan is updating its fire fighting provision. Fire hydrants are to be placed so that that every block can be reached in the event of a fire.

What is the minimum number of hydrants and how should they be arranged?

Allow pupils to discuss what information they need, any possible problems, and so on.

Set up a simplified mathematical model and constraints.

- The blocks are 100m square.
- Fire hoses extend to 100m and need only stay at ground level.
- Hydrants have to be placed at street corners.

Explain that modelling is a way of exploring the overarching problem and finding possible solutions. Any solutions must be checked against the initial problem. When applying solutions to a 'real context', specific issues also need to be taken into account. For example, in the case of downtown Windermere there are some extra streets not part of a rectangular grid, some blocks appear to be larger than others, and so on.

Allow pupils to discuss possible approaches. You could direct discussion towards starting with a small section of the town, such as a 4 x 2 grid.

Pupils could use the flash interactive, mini-whiteboards, multilink cubes or square tiles and pipe cleaners to investigate the problem, but they should remember to record their results.

Pupils can also use the Flash interactive to present their findings to the whole class.

During the activity

Encourage pupils to explore their ideas and to make their own decisions about what aspects of the problem they would like to investigate.

Allow pupils to develop their own approach to recording results.

When pupils are consistently generating minimum results, encourage them to explore a particular group of grids.

Suggest to pupils that they collect a number of results before making conjectures about the minimum number of hydrants required.

Where pupils make predictions, encourage them to make sense of their conjectures as well as testing them.

If pupils find an algebraic rule, encourage them to justify it within the context of the original problem.

Probing questions

AMP activities are well suited to formative assessment, enabling pupils to discuss their understanding and decide how to move forward. See www.nuffieldfoundation.org/whyAMP for related reading.

- What are you trying to find out?
- How do you know you have found the minimum number of fire hydrants needed for your grid?
- How will you decide if your prediction is correct?
- You have found a rule. Why do you think your rule will always work? What does your rule tell you about the original problem?

Extensions

- Using fire hoses of different lengths, such as 200m or 50m.
- Using city blocks of different shapes, such as rectangles, regular hexagons, equilateral triangles.
- Exploring different arrangements of a fixed number of blocks, say 100, to determine which arrangement will need the smallest number of hydrants.

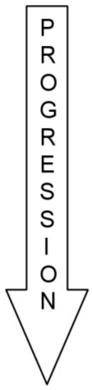
Progression table

The table below can be used for:

- sharing with pupils the aims of their work
- self- and peer-assessment
- helping pupils review their work and improve on it.

The table supports formative assessment but does not provide a procedure for summative assessment. It also does not address the rich overlap between the processes, nor the interplay of processes and activity-specific content. Please edit the table as necessary.

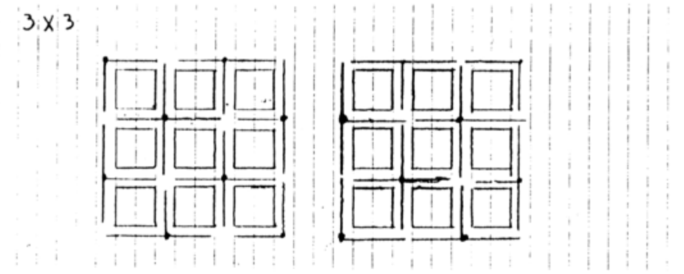
Representing *Selecting a suitable mathematical approach and deciding how to record results*	**Analysing** *Accurate results with sufficient detail to work towards a general solution*	**Interpreting and Evaluating** *Identifying and explaining patterns and exceptions; making and justifying generalizations*	**Communicating and reflecting** *Explaining the approach taken and outcomes achieved at each stage*
Shows understanding of the task Pupil A	Finds minimum number of hydrants required for a chosen grid Pupil A	Makes simple observations, indicating why the number of hydrants is the minimum Pupils A, B	Uses diagrams to communicate observations Pupil A
Produces a series of diagrams to explore arrangements of hydrants in particular families of grids Pupil B	Brings together findings Finds simple numerical patterns and identifies possible exceptions	Makes valid comments about the different patterns and relates these to the geometry of the grid Pupil C	Organises examples to illustrate findings
Works systematically to explore different grids within a clearly identified family Pupil D	Takes systematic approach to generating results Pupils B, D	Explains how suggested arrangements of hydrants can be extended for related grids Pupil D	Variables are explicitly defined Any diagrams or graphs are suitably labelled Pupils D, E
Uses diagrammatic and other abstract approaches to find the minimum number of hydrants needed for a grid of any given size	Organises investigation so that several different factors are analysed efficiently	Finds a general rule and justifies it within the context of the original problem	Describes clearly what was investigated, and explains how conclusions were arrived at



Sample responses

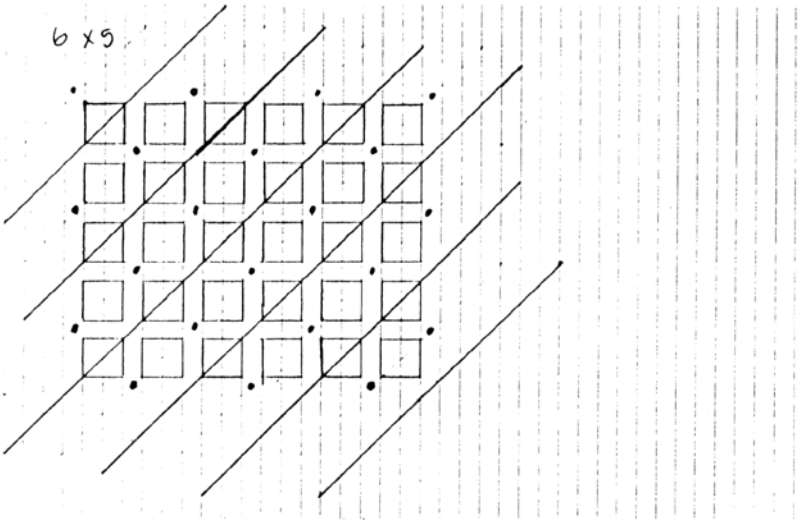
Pupil A

Pupil A has explored the overall picture and found the minimum number of hydrants needed for 3 x 3 and 6 x 5 grids. The observation has been made that hydrants lie on alternating diagonal lines.



The Hydrants are placed in diagonal lines across the blocks, alternating with and without Hydrants.

The diagram below shows.



Probing questions

- Are the two arrangements for hydrants on a 3 x 3 grid the same?
- Does putting hydrants on alternating diagonal lines always use the minimum number of hydrants?

Pupil B

Pupil B has chosen to explore different arrangements of 36 blocks, tabulating results, and has identified an optimal arrangement.

Number of blocks = 36

Grid Size	Number of hydrants
1 x 36	37
2 x 18	28
3 x 12	26
4 x 9	25
6 x 6	24

The most effective arrangement is 6x6 where only 24 hydrants are needed.

Probing questions

- Why is 6 x 6 the most effective arrangement of 36 blocks? How is it different from other arrangements of 36 blocks?
- Does you result tell you what might be the most effective arrangement of other numbers of blocks?

Pupil C

Conclusions

The closer the two factors are (eg 4 x 6, 6 x 6) the less hydrants are needed. If the two factors are far apart (eg. 1 x 24 and 1 x 36 the more ~~effech~~ hydrants are needed.

This suggests that for 100 city blocks the most effective arrangement with the least number of hydrants will be 10 x 10.

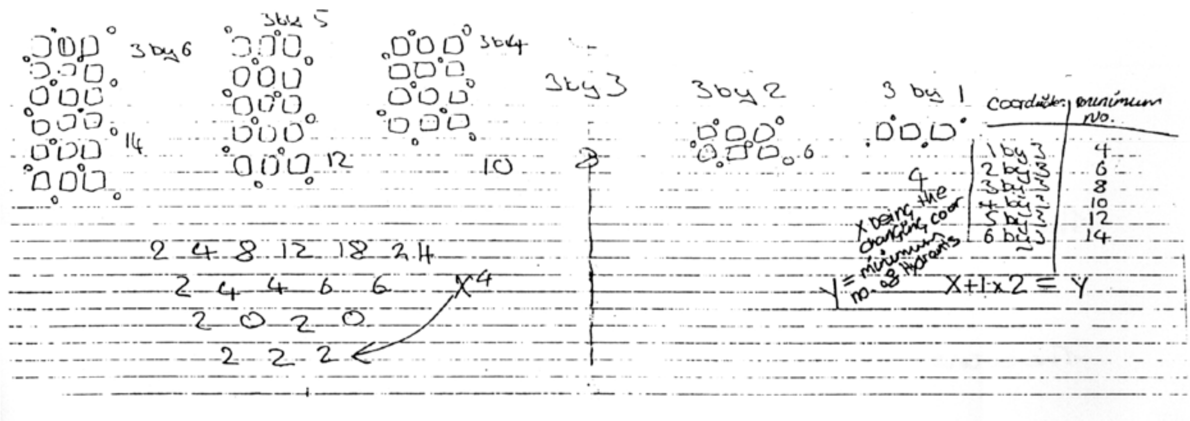
Pupil C has investigated rectangular grids and made a valid generalisation about the most efficient arrangement of blocks. The specific numbers of hydrants needed are not given, and no further justification or interpretation is offered for the conclusion.

Probing questions

- Why may your suggestion be correct?
- You've made a conjecture about which arrangement of 100 blocks will need the smallest number of fire hydrants. Do you what that number is, or how the hydrants will be arranged?

Pupil D

Pupil D has drawn arrangements of the minimum number of hydrants



for grids of the form 3 x *n*. Correct results have been tabulated, but an incorrect rule has been given for the minimum number of hydrants for these grids. No interpretation of the rule has been offered.

A set of results for the minimum number of hydrants needed for square arrangements has been listed, and the pupil has attempted to produce a difference table. No indication is given of how these results were obtained.

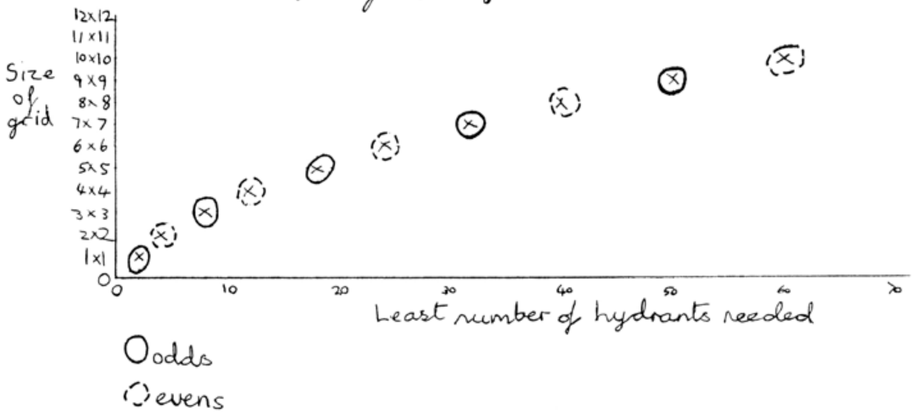
Probing questions

- Does your rule agree with your results?
- Why does your rule work?
- You have explored grids with a width of 3 and variable length. What do you think may happen if the width of the grid also varies?

Pupil E

Pupil E has explored square grids and has chosen to summarise the results in a sketch graph, highlighting the difference between odd and even numbers of blocks. The pupil has stated two distinct algebraic expressions for the number of hydrants needed, one for the odd case and one for the even case, but believes it is possible to find a single formula. There is no evidence of how the expressions were found.

Now I am going to see if I can make up an algebraic formula which can be used to find out how many hydrants would be needed for any square grid.



Because all the crosses are on one curve there is obviously an equation that covers both odd and even. I can only find one for each.

N = number of blocks down one side of the grid

The odds equation is; $(N+1)\left(\frac{N+1}{2}\right)$

The evens equation is; $\frac{N}{2}(N+2)$

Probing questions and feedback

It would be worth having a discussion with the pupil about the accuracy of their graph, and why they feel having two separate formulae is inadequate.

- How did you obtain your formulae? Do they tell you anything about the arrangement of the hydrants?
- Why did you choose to explore a square arrangement of blocks? What do you think may happen if the arrangement of blocks is not square?