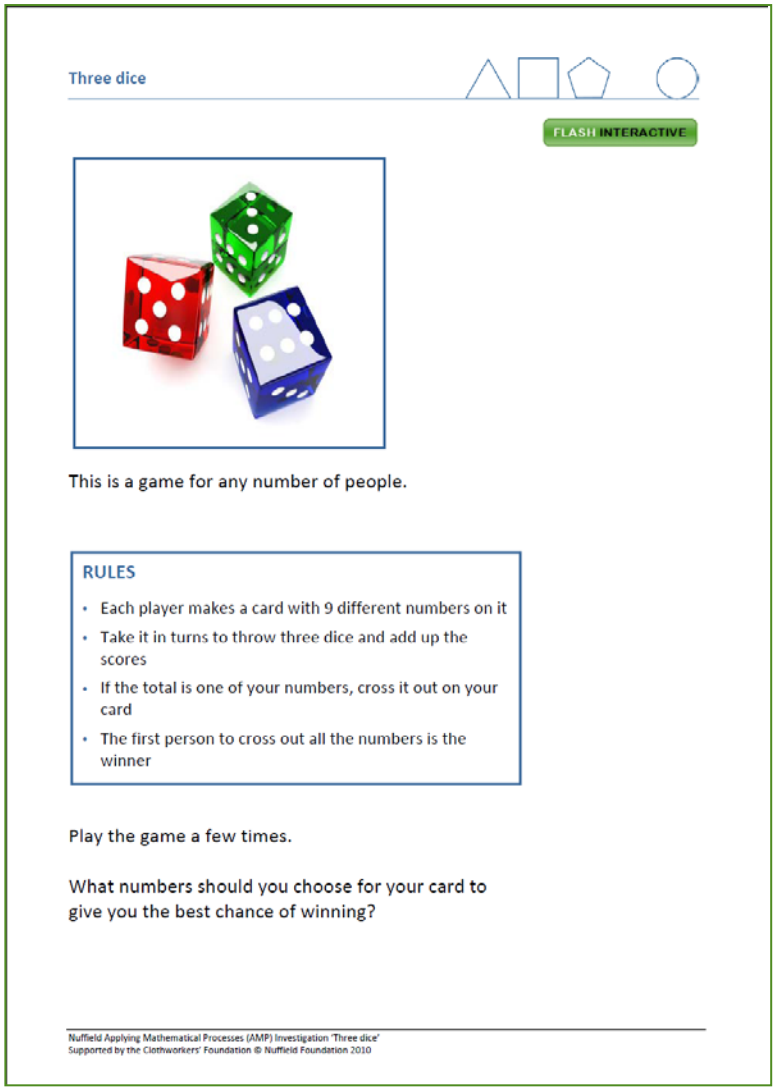
Activity description

This activity starts with a game. In order to maximize their chances of winning the game, pupils must decide which numbers are most likely to occur when three dice are thrown and the scores are added together.


Three dice

FLASH INTERACTIVE

This is a game for any number of people.

RULES

- Each player makes a card with 9 different numbers on it
- Take it in turns to throw three dice and add up the scores
- If the total is one of your numbers, cross it out on your card
- The first person to cross out all the numbers is the winner

Play the game a few times.

What numbers should you choose for your card to give you the best chance of winning?

Nuffield Applying Mathematical Processes (AMP) Investigation 'Three dice'
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Suitability

Pupils working at all levels; individuals or in groups

Time

1 – 2 hours

AMP resources

Pupil stimulus, card grid and Flash interactive

Equipment

3 differently-coloured dice for each pair or small group (differently-coloured or distinguishable if possible)
Dice simulator
Spreadsheet facility

Key mathematical language

Sum, outcomes, frequency, combinations, permutations, most/least likely, trials, probability, theoretical probability, experimental probability

Key processes

Representing Identifying the mathematics involved in the task, and developing an appropriate representation.

Analysing Working systematically, identifying patterns, making generalisations.

Interpreting and evaluating Considering findings and forming conjectures; justifying findings; considering validity of theoretical model.

Communicating and reflecting Explaining approaches taken at each stage of the work; considering experimental and theoretical models.

Teacher guidance

Play the game as a whole class. The dice simulator or a random number generator on a computer could be used. Further information on the dice simulator is given below.

Initially allow pupils to select any 9 numbers.

Ensure that pupils understand the game.

After playing the game once or twice, allow pupils to refine their choice of numbers and ask for reasons for their choices. Consider playing the game again as a lesson plenary, and ask pupils to reflect on their choice of numbers in the starter and plenary games.

Encourage the development of strategies to find out why some numbers are better for the game than others. Pupils could start to analyse the equivalent game with only two dice and a bingo card of six numbers as a first step towards understanding the activity.

During the activity

Encourage pupils to use any intuitive feeling they have for the problem.

To enable pupils to progress beyond the initial 'playing bingo' stage, encourage careful recording of the numbers they use in their games and the numbers displayed on each die for each throw. Using easily distinguishable dice, such as differently coloured dice, may help here.

Ask pupils to record and explain any observations they make about their experimental data.

Pupils may need reminding that their findings, conjectures, and the results of testing their cards need to be communicated clearly.

Encourage pupils to move from collating experimental data to working with a theoretical model, considering which combinations are possible for different totals and the number of ways in which each combination can occur.

Again, using three distinctly different dice will help to highlight the different permutations of each combination of numbers. For example (1, 1, 2), (1, 2, 1) and (2, 1, 1) all give a total of 4.

Probing questions

AMP activities are well suited to formative assessment, enabling pupils to discuss their understanding and decide how to move forward. See www.nuffieldfoundation.org/whyAMP for related reading.

- When you throw the three dice several times, do some numbers come up much more often than others?
- Which totals do you think are the most/least likely? Why?
- How many different ways can each total be made?
- Is there anything you notice that is similar about the ways of making 3 and 18, or 4 and 17? Can you explain this?

Extensions

- What numbers would you put on your card if you were to use octahedral dice (numbered 1 to 8), or decimal dice (0 to 9)? How are you using your results from the original investigation to inform your choices?
- If pupils were allowed to repeat numbers on their cards, but only cross out one copy each time, how would this affect their choice of numbers for a winning card?
- If only two dice are thrown, the possible scores can be presented on a 6x6 grid. Pupils could try to represent the equivalent for three dice on a 6x6x6 cube. It might be easier for pupils to visualize the cube as six 6x6 layers, rather than constructing it completely. Multilink cubes could be used, and it would be an interesting exercise for pupils to decide how to allocate the colours. For example, they might use only a few colours and group scores according to their likelihood.
- What would happen if your dice weren't fair? Pupils could model this problem by making spinners with different areas for different scores to weight the outcomes.

Dice simulator

The included dice and spinner simulator has been made available thanks to NRICH http://nrich.maths.org/6717

Click on the dice button (top of screen) three times to produce three dice, which can be dragged around the screen if necessary.

Pressing the play button simulates one throw of the dice.

Clicking the results button produces a frequency table for the total scores. The graph tab shows the same results in a bar chart.

The results are updated each time you press play.

The down arrow next to the play button gives options for throwing the dice 100 or 1000 times.

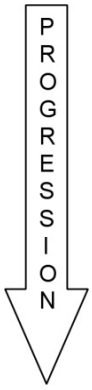
Progression table

The table below can be used for:

- sharing with pupils the aims of their work
- self- and peer-assessment
- helping pupils review their work and improve on it.

The table supports formative assessment but does not provide a procedure for summative assessment. It also does not address the rich overlap between the processes, nor the interplay of processes and activity-specific content. Please edit the table as necessary.

Representing *Clear choice of approach and appropriate form of representation*	**Analysing** *Accurate results with sufficient detail to work towards a solution* *Identifying patterns*	**Interpreting and evaluating** *Explaining or justifying findings and generalizations* *Considering validity of theoretical model*	**Communicating and reflecting** *Clear explanations of how results were found and how they led to overall conclusions*
Shows understanding of the task Records totals scored from the three dice	Obtains some results, for example by rolling 3 dice, and crosses off any totals appearing on their card	Modifies numbers on cards to include more 'successful' numbers Pupil A	Makes an observation such as simple statement, identifying some winning numbers Pupils A, B
Chooses to record the frequencies of different total scores	Identifies numbers occurring more or less frequently in trials, and suggests reasons for this Pupils A, B	Explains why some numbers occur more or less frequently than others Pupil B	
Uses a systematic method to collect trial data Pupil C	Calculates relative frequencies, percentages, or experimental probabilities of different totals Pupil C	Interprets relative frequencies, percentages, or experimental probabilities by choosing numbers for a winning card and testing it	Explains why they have chosen the numbers on their winning card Comments on results of tests
Moves from recording experimental data to developing a theoretical model Pupils D, E, F	Lists all possible combinations of scores to make each total Incorporates permutations or probabilities into reasoning Pupil D, E, F	Interprets some theoretical results and provides justification; uses theoretical model to devise a winning card and tests it Pupil D	Uses a variety of forms to present results Connects experimental results with theoretical solution Pupil F
Presents an overall approach to the problem, e.g. devises a model for representation that illustrates all possible outcomes	Deduces theoretical results using permutations, symmetry, etc Calculates correct theoretical frequencies or probabilities for all outcomes	Uses theoretical probabilities to choose numbers for winning card and tests card	Comments on differences between theoretical and experimental results

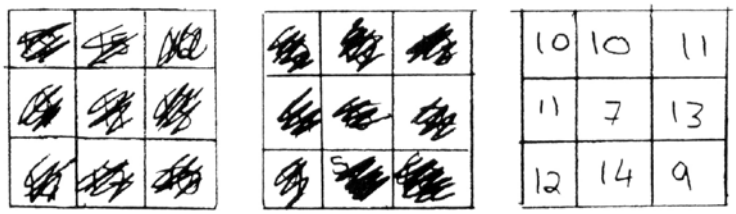


Sample responses

Pupil A

Pupil A has written down several sets of 9 numbers, including repetitions, and crossed them off as in *Bingo*.

The pupil has observed that certain totals occur more often than others and has chosen to use 10 and 11 twice in his third card.



10 and 11 came very, very often
15 and 5 came very rarely.

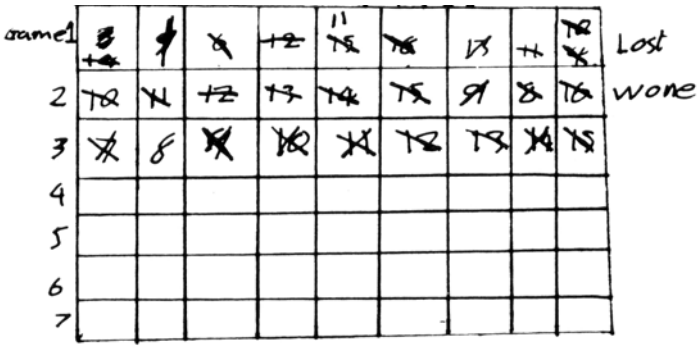
Probing questions

- You say that 10 and 11 came up 'very very often'. How many times did they occur?
- You say that 5 came up very rarely. How many ways are there of making a total of 5 with 3 dice? Is that different from the number of ways of making 10?
- I notice that 8 is not on your third card, but 7 is. How did you decide this?

Pupil B

Pupil B has not recorded the results of the dice throws, so it is not clear how he is choosing his numbers for each game.

Some numbers are identified as occurring more or less often and the pupil suggests a reason for this, but it is not illustrated or justified.

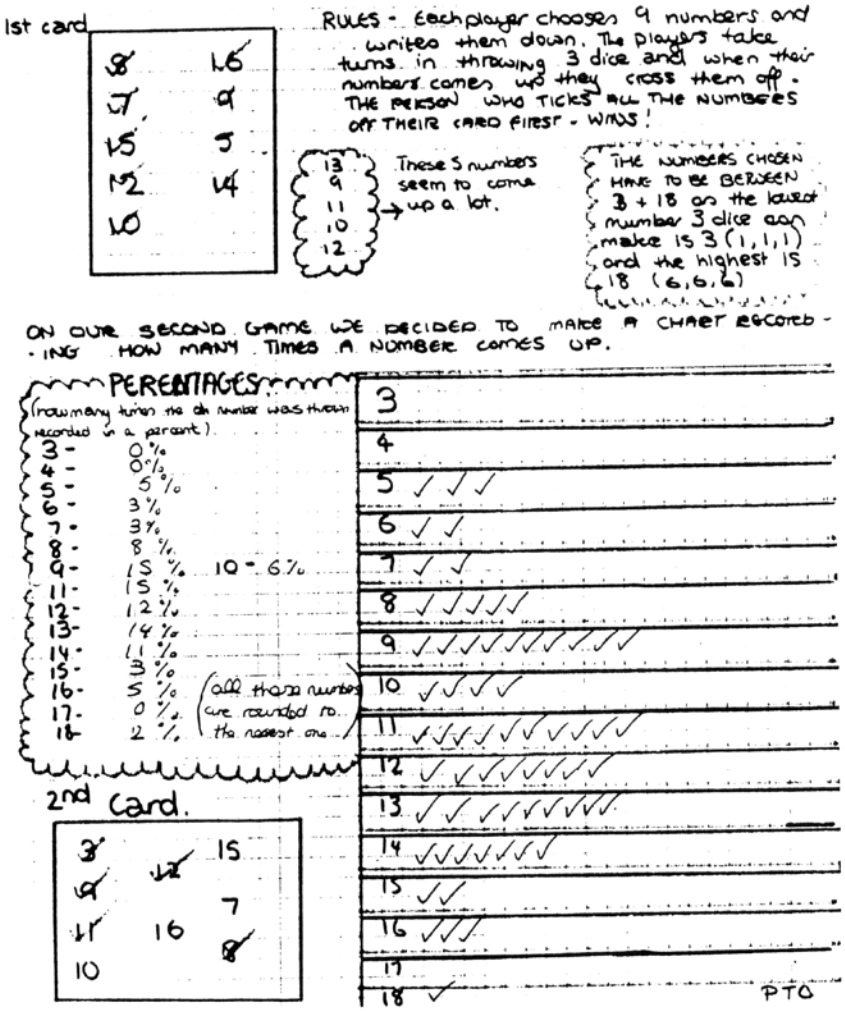


Number 11 10 12 and 13 keep coming up because thair are lots of ways to = them but 3 4 17 and 18 Hardley ever come up because ~~th~~ thair are not many ways to = them.

Probing questions

- Would recording the dice scores help you to choose numbers for a winning card?
- How many ways are there to make the numbers that you say are hard to make?
- You say that 10, 11, 12 and 13 keep coming up because there are lots of ways to make them. Can you work out how many ways exactly?

Pupil C



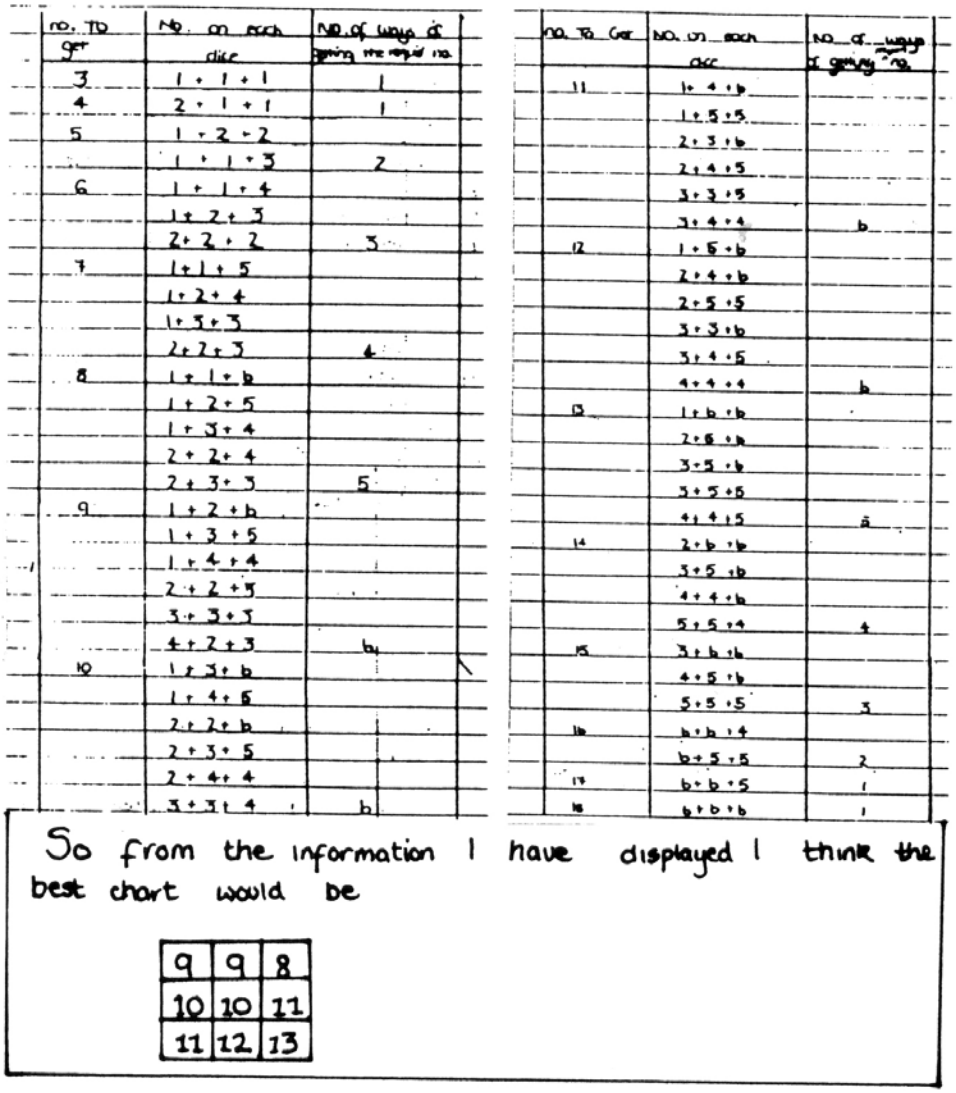
For the second attempt at the game, Pupil C has recorded results of the dice throws systematically and used these to find percentages for each possible score.

Pupils C has also identified how 3 and 18 could be scored, but has not related this to the experimental results

Probing questions

- You say there is only one way of making 18, and only one way of making 3. How many ways are there of making other numbers? Is this connected with the results you recorded?
- How could your findings help you choose numbers for a winning card?

Pupil D



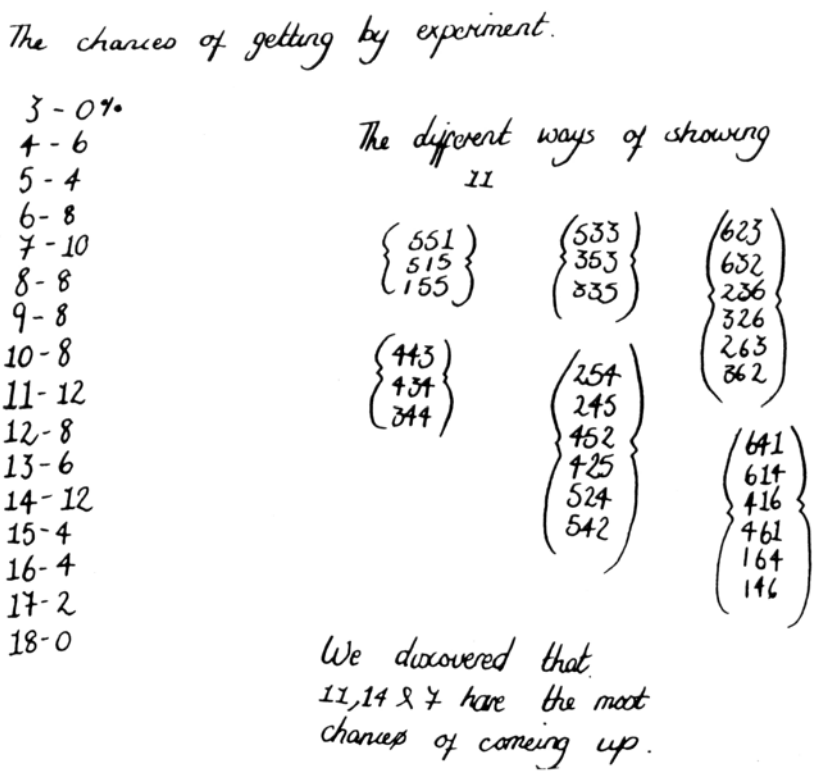
Pupil D has listed all possible combinations of scores to make each total, but has not taken into account the different ways the numbers can occur on the three dice.

Pupil D was playing a form of the game where repetition of numbers is allowed. The pupil claims that the information he has displayed led him to choose the numbers given on the card, but no reasoning is given, either for the choice of numbers, or the number of repetitions of each.

Probing questions

- You have listed one way of making 4 and one way of making 3. Do you think this means that both totals are equally likely?
- You list 1,2,3 as a way of making a total score of 6. Does it matter which die shows which number?

Pupil E

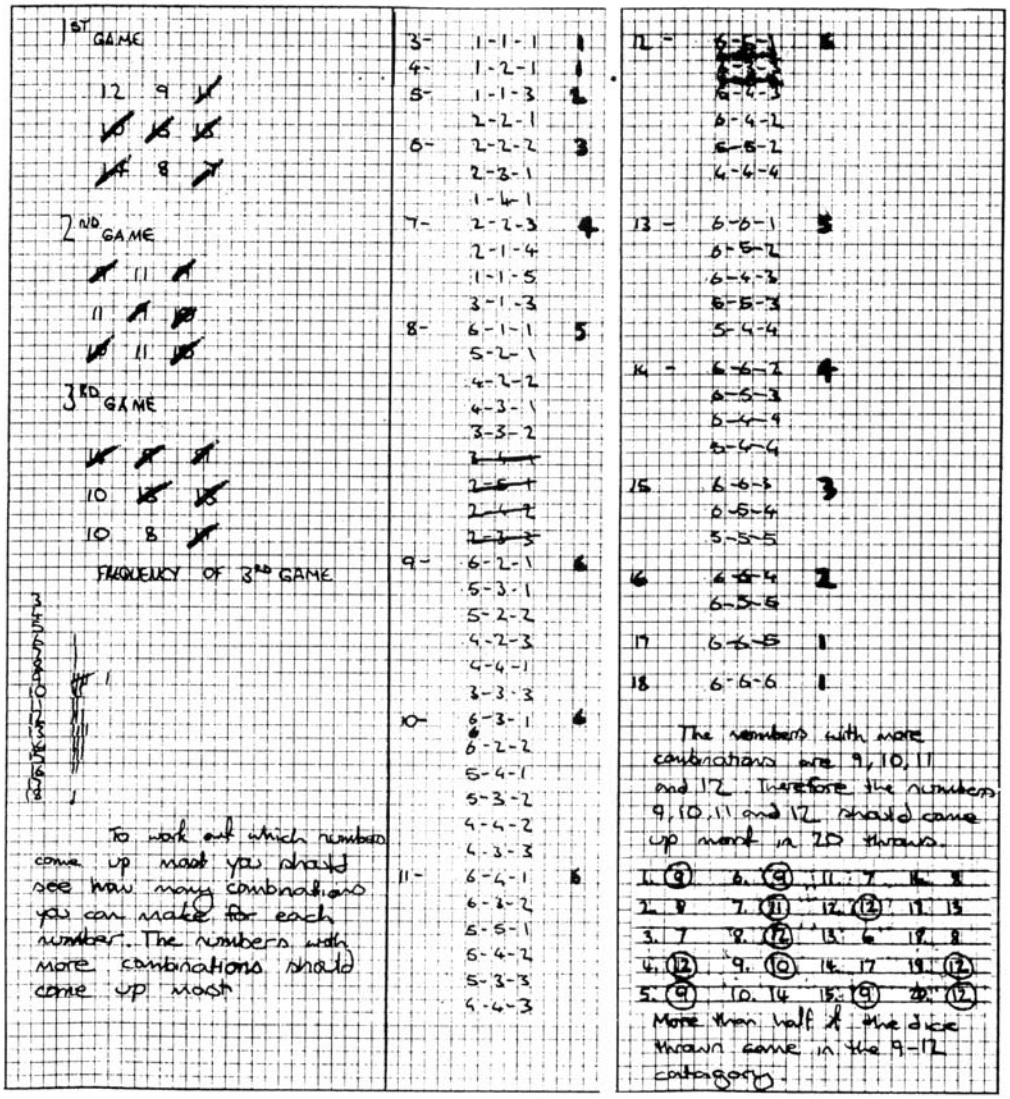


Pupil E has shown experimental results for all scores and commented on these results, but has not attempted to explain them. The pupil has also found all the ways of producing a total score of 11. There is no evidence that the pupil has made a connection between the two aspects of their investigation.

Probing questions

- Tell me why and how you found 27 different ways of making 11.
- In your experiment, 14 came up the same number of times as 11. Do you think 14 and 11 are equally likely to occur? Are there also 27 ways of making 14?
- I notice in your experiment that 3 and 18 are each 0%. Do you think that 3 or 18 will never appear?
- The numbers 8, 9, 10 and 12 in your trials have equal frequencies. Can you find the number of ways of making each of these numbers?

Pupil F

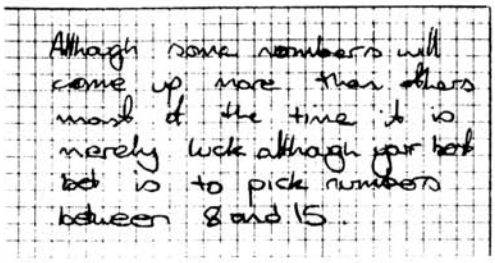


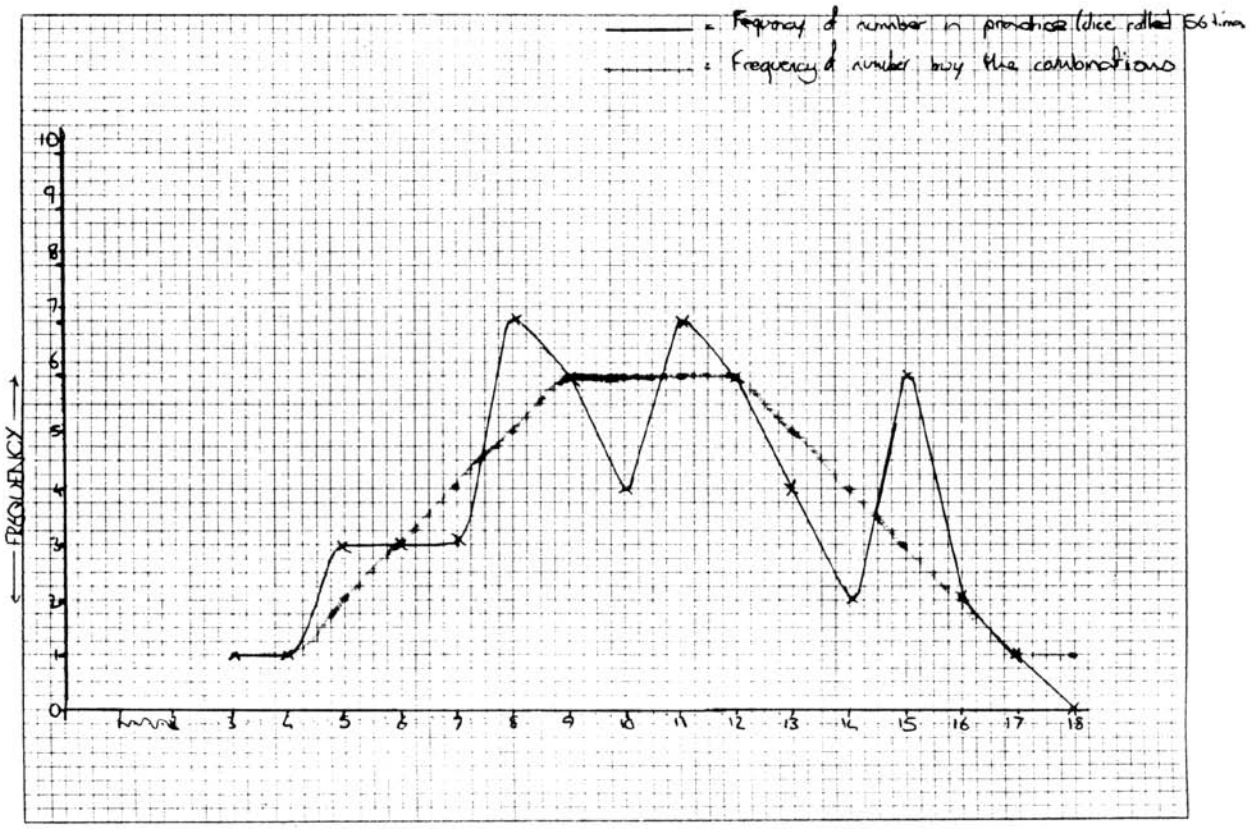
Pupil F has observed a connection between the number of combinations that make a certain total and the number of times that total will occur.

All combinations of dice have been recorded systematically, but different permutations have not been counted. Theoretical and experimental results have been compared graphically. Pupil F has suggested a winning strategy of choosing numbers between 8 and 15.

(Pupil F continues on the next page)

Pupil F *continued*





Probing questions

- I notice that, for a total of 8, you did some work which you then crossed out. Can you tell me why you did this?
- Is 4 – 2 – 2 the same as 2 – 4 – 2? Is 2 – 5 – 1 the same as 5 – 2 – 1? Does the arrangement of the numbers over the three dice affect the frequency of the total?
- How many possible outcomes are there if you throw three dice? Can you account for all of these?