Towards universal participation in post-16 mathematics: lessons from high-performing countries

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Other publications accompanying this report are available online at www.nuffieldfoundation.org/towards-universal-participation.

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A technical report with full details of the methodology, including how the participation rates were calculated.

In-depth profiles of each of the countries featured in the study.

Foreword

In December 2010, the Nuffield Foundation published *Is the UK an outlier? An international comparison of upper secondary mathematics education*. This report revealed that England, Wales and Northern Ireland have the lowest levels of participation in post-16 mathematics out of 24 countries surveyed. They were the only countries in which fewer than one in five students in this age group studied mathematics. Scotland's participation rate was higher, but still below average.

The report has had a substantial impact. Its findings have been widely referenced – by government, and by education and business leaders – as evidence for the need to reform post-16 mathematics education. We therefore welcomed the new goal proposed by the Secretary of State for Education in June 2011 that "within a decade the vast majority of pupils are studying mathematics right through to the age of 18". Since then the Department for Education, the Advisory Committee on Mathematics Education (ACME), the Nuffield Foundation and others have engaged in a range of developments to explore how this aspiration might best be achieved. In its two papers published in December 2012, ACME made a series of recommendations for increasing provision and participation in post-16 mathematics, one of which was the development of a new qualification.

While there is now a broad consensus that change is necessary, the debate about the precise nature of this change will continue, particularly in the context of concurrent reforms of GCSEs and A levels. At the Nuffield Foundation, we are keen to ensure this debate is informed by rigorous and independent evidence. We also maintain the view that there are important lessons to be learned from policy and practice in other countries. To this end, we commissioned the principal author of *Is the UK an outlier?*, Professor Jeremy Hodgen of King's College London, to lead a further international study looking in greater depth at a smaller number of high-performing countries. Our aim was to identify the factors related to high levels of participation in post-16 mathematics and their implications for the UK.

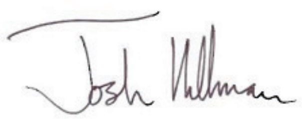
As the authors note, international comparisons and the transfer of ideas from country to country are inherently problematic. Education systems are embedded within the specific cultural and political context of each country, and measurement is never like-for-like. Cherry-picking specific features can be dangerous, especially when moving from compact, homogeneous systems to more diverse ones. However, with these caveats, there are some findings that have the potential to inform both short- and long-term developments in the UK.

A key message is that high levels of participation are not simply driven by compulsion, particularly for advanced mathematics. Other factors, such as the availability of appropriate pathways and the breadth of the post-16 curriculum generally, are associated with high levels of participation. Where mathematics is compulsory, it is never the only compulsory subject. This is important, because it tells us that we cannot look at mathematics or compulsion in isolation; they need to be considered within the wider curriculum and qualification framework both pre- and post-16.

Another strong message is the importance of a collaborative and holistic approach to policy development, including around the issue of teacher capacity. We believe there is a danger of underestimating the challenges for teacher recruitment, deployment and development posed by a rapid expansion of post-16 mathematics education. To address this, we will be looking to improve the evidence base in this area and to encourage the development of new practical ideas.

The evidence from this study suggests that the education system in England is unusual in its complexity, including in the range of qualifications available. Yet there is limited take up of mathematics and statistics at an advanced level. The experience of other countries, particularly New Zealand, shows that it is possible to increase participation by providing an alternative pathway, focused on statistics, that is widely recognised and valued by higher education and employers. We are particularly interested in this finding, given our work promoting statistical literacy and our new programme to promote a step-change in quantitative methods training for social scientists. Indeed, we believe there is much to learn from the New Zealand model, and hope it will inform the development of an attractive and valued qualification in this country.

We are grateful to Jeremy and his co-authors, Rachel Marks and David Pepper, who have undertaken this important piece of work with commitment and incisiveness. It will provide useful evidence for current and future policy development. Ultimately, we hope it will contribute towards many more young people gaining the mathematical and statistical skills necessary for further and higher education, employment, and active participation in modern life.



Josh Hillman, Director of Education

11. Introduction

This study is a follow-up to the Nuffield Foundation report *Is the UK an outlier? An international comparison of upper secondary mathematics education* (Hodgen et al., 2010). The report examined mathematics education provision in 24 different countries and revealed that England, Wales and Northern Ireland have the lowest levels of participation in upper secondary mathematics. They were the only countries in which fewer than 20% of students studied mathematics at upper secondary level. Scotland had a higher participation rate than elsewhere in the UK, but this was still below average.

This study aims to build on the participation data in the previous study by looking in more detail at a smaller number of countries. These countries are England, Germany (Rhineland-Palatinate), Hong Kong, New Zealand, Scotland, Singapore and the United States (Massachusetts).

The study is based on two research questions:

1. What factors drive participation in upper secondary mathematics?
2. What is the content and level of upper secondary mathematics provision and how does it vary across different general and vocational routes?

The research was carried out in three stages:

1. Academic literature search
2. Compilation of detailed country profiles
3. Review and synthesis of available data

The meaning of 'upper secondary' in this report

In the countries included in this research, 'upper secondary' is usually between two and four years in duration and intended for 15/16-year-olds to 18/19-year-olds. It includes all forms of education for this age group, including school, college and employment-based options, but excludes tertiary education. In the UK, upper secondary education is commonly referred to as 'post-16', but we chose not to use this term for reasons of comparability between countries. For example, in several countries upper secondary begins at a younger age than 16. Furthermore, many countries require students to repeat grades, meaning that unlike the UK, not all post-16 students are in upper secondary education.

The meaning of 'country' in this report

We use the term 'country' to describe the different education systems included in the study, although for Germany and the USA, we have only looked in detail at one state or land, and Hong Kong is a special administrative region of China rather than a country. We have done this so that the terminology is consistent with the previous study and because we needed a simple description that could be used across what is actually a group of different legal, political and social entities.

2. Key findings and recommendations

Our evidence is strongest on describing what countries do, and weakest concerning the causal factors underlying participation levels in advanced mathematics. While we found some literature addressing the issue of causation, there were considerable gaps in the research evidence. In addition, the English education system was, aside from the USA as a whole, the most complex that we examined. Consequently, we caution that policies and strategies from other countries would be likely to play out very differently in England.

Policy development and implementation always carries a risk of unintended consequences. For example, the introduction of AS levels resulted in a very large drop in the number of students taking A level Mathematics in 2002, and recovery to previous levels of participation took almost a decade. So while we make suggestions for how approaches from other countries might influence strategies for increasing participation in mathematics in England, we also identify some potential risks that could arise as a result.

1. Factors in increased participation in upper secondary mathematics

- Making mathematics compulsory in upper secondary education is a critical factor in achieving universal participation in basic mathematics, but is not associated with the highest levels of participation in advanced mathematics.
- Some countries still achieve very high (but not universal) levels of participation in basic mathematics without compulsion.
- Mathematics is never the only compulsory subject in upper secondary education.

Hong Kong, the USA and Germany all achieve near universal participation in at least basic mathematics. Mathematics is compulsory in upper secondary in Hong Kong and the USA, and in Germany is compulsory in almost all general and vocational pathways. However, these countries do not have the highest levels of participation in advanced mathematics. The introduction of compulsion in mathematics in Hong Kong appears to be associated with a slight reduction in participation in advanced mathematics.

New Zealand and Singapore, the countries with *the highest levels of participation in advanced mathematics*, achieve very high rates of participation in basic mathematics without compulsion for all students. However, in New Zealand, students are required to study mathematics until they have achieved a basic level of competence (NCEA Level 1). Singapore's extended basic mathematics course gives some students an extra year to master the content and develop fluency. This stands in marked contrast to the increased use of early entry in England, whereby students take GCSE Mathematics a year or even two years early, and the remedial focus of some current GCSE re-sit courses.

In our earlier study (Hodgen et al., 2010), we found that mathematics is never treated as a special case. Where mathematics is compulsory, at least one other subject is also compulsory. Indeed, generally both the national language(s) and mathematics are treated as core subjects. Requiring a broader range of subjects affects the amount of curriculum time devoted to each subject and the depth of content studied.

Recommendation 1: All students in England should be enabled to study mathematics in upper secondary education at an appropriate level.

Recommendation 2: GCSE Mathematics should remain compulsory until students have attained a Grade C as recommended in the Wolf Report (2011). However, the GCSE syllabus should not be curtailed through early entry. Consideration should be given to enabling students to prepare for GCSE for an additional year or more in order to avoid a remedial focus and to afford the best chance of mathematical fluency.

Recommendation 3: Any plans to make upper secondary mathematics compulsory in England should also consider the subject status of English language. The breadth of student programmes and the effects on other subjects at advanced level should also be considered.

2. Appropriate mathematics qualifications in upper secondary

- **The availability of appropriate qualifications in advanced mathematics is crucial to increasing participation.**

- **In New Zealand, the widespread availability of a well-respected option focused on mathematical fluency, statistics and the application of mathematics appears to have increased participation in advanced mathematics.**

- **For many disciplines in higher education, mathematical fluency is more important than knowledge of calculus.**

Mathematics qualifications in England are more complex than in most other countries. Despite this, there is no qualification that is *widely available* to students who have achieved a grade C at GCSE but who do not wish to choose a traditional mathematics with calculus option (see Appendix A for a comparison of pathways).

In New Zealand, students are offered a choice of options in two areas of advanced mathematics: mathematics with statistics, and mathematics with calculus. They can take options from both areas. The availability of advanced mathematics options focused on statistics and modelling appears to be a key factor in increasing participation to one of the highest levels internationally. These options are widely available, attract a critical mass of students and are widely recognised by higher education and employers.

While advanced mathematics with calculus provides a good preparation for higher education courses in engineering, physics and mathematics, emerging evidence suggests that it is not a necessary preparation for disciplines such as biosciences, geography, or business and management, where fluency and statistics may be more relevant (see for example Advisory Committee on Mathematics Education, 2011; Koenig, 2011). However, further research is needed, particularly about the mathematical needs of employers.

Recommendation 4: Introduce *one* new advanced mathematics pathway aimed at those who have achieved a grade C or above at GCSE but are not currently studying advanced mathematics. The assessment should be carefully designed to focus on fluency, modelling and statistics. Where possible this should build on the best elements of existing qualifications such as FSMQs.

Recommendation 5: Research is required about the mathematical needs of employers, including what advanced mathematics and modelling skills are needed, and how these vary across different occupations.

3. Breadth of subjects

- **Requiring or encouraging students to take a breadth of subjects is associated with increased participation in advanced mathematics. This may be more effective than compulsion in increasing the study of advanced mathematics.**

The two countries with the highest participation in advanced mathematics, New Zealand and Singapore, allow students a choice of subjects at advanced level but also require breadth in these choices. In New Zealand, students are encouraged to take modules in eight or nine subjects. Students in Singapore are required to choose a contrasting subject equivalent in size to an AS level, so arts and humanities students must take a science or mathematics option and vice versa. Although participation is not as high as in New Zealand and Singapore, Scotland requires students to study five or more subjects at advanced level and has participation levels in mathematics that are considerably higher than in England.

The changes to upper secondary mathematics in New Zealand and Singapore have resulted in a reduction in participation within some advanced mathematics options, although overall participation in advanced mathematics has increased in both countries. Singapore's requirement to take a contrasting subject has led to a significant decrease in participation in further mathematics because students now choose a subject that contrasts with advanced mathematics. In New Zealand, the introduction and popularity of the mathematics with statistics option has led to reductions in participation in the mathematics with calculus option.

Recommendation 6: Consideration should be given to encouraging a greater breadth of advanced study. This should be through mechanisms that are simple and straightforward, such as those in New Zealand and Singapore.

Recommendation 7: Consideration should be given to mechanisms and incentives aimed at ensuring that existing advanced qualifications (including AS and A level Mathematics and Further Mathematics) maintain their existing levels of participation if and when a new advanced mathematics pathway is introduced.

4. Prior attainment, attitudes, advice and incentives

- Prior attainment is a crucial factor in continued participation in mathematics, particularly at an advanced level.
- Attitudes to mathematics appear to be less important than prior attainment but the former area is less well evidenced.
- There was limited evidence on the efficacy of information, advice and guidance available to students. However, making students aware of the personal rewards and benefits of advanced mathematics qualifications can lead to increased participation.

- **The experience of Hong Kong, New Zealand and Singapore indicates that the strongest incentive for students to study advanced mathematics is that they are required to do so to progress to higher education and employment.**

Prior attainment is key to continued participation in advanced mathematics, but improving attainment across an educational system is very difficult to achieve. Furthermore, high attainment alone is not enough. For example, New Zealand's mathematical performance is only slightly above average (PISA 2009), but it has a much higher level of participation in advanced mathematics than Hong Kong, one of the highest performing countries on international surveys. New Zealand's option of an attractive alternative qualification in advanced mathematics that *does* include statistics but *does not* involve formal calculus appears to be an important factor.

Although positive attitudes and aspirations are associated with increased participation, there is very little evidence to indicate whether and how attitudes and aspirations might be causally related to increased participation. Recent research from England suggests that providing students with convincing information on the extrinsic benefits of the subject, particularly where supported by strong relationships with teachers, is an important factor in raising participation in advanced mathematics (Mujtaba, Reiss, Rodd & Simon, in preparation). However, we found little evidence to indicate that improving attitudes and aspirations alone is likely to lead to significant increases in participation and attainment. We also found limited evidence on the nature and efficacy of the information and guidance available to students.

Hong Kong's move to compulsory mathematics as part of a new upper secondary qualification has been accompanied by a slight decrease in participation in advanced mathematics, partly because there are few incentives to take these options (and in particular many highly numerate higher education courses in Hong Kong universities do not require advanced mathematics). Conversely, in Singapore, a key factor appears to be the requirement for advanced mathematics for many degree courses. While caution needs to be exercised in drawing causal conclusions from a small number of countries, this does suggest that higher education requirements are important.

Recommendation 8: The needs of higher education and employers should be taken into account in the development and implementation of any new advanced mathematics pathway, both in relation to its content, and also to enable it to be built into the requirements of relevant higher education courses and employers' recruitment and selection processes. In addition, any new pathway should be attractive to students who do not currently take AS or A level Mathematics.

Recommendation 9: Higher education and employers should require some study of mathematics beyond GCSE for study or employment in numerate disciplines including social science, teaching and nursing.

Recommendation 10: Further research is needed to evaluate the effects – and cost benefits – of specific approaches to giving information, advice and guidance to students, particularly those interventions aimed at informing students about the personal rewards and benefits of studying advanced mathematics.

5. Stratification and the role of gender and socio-economic status

- **Separating students into different pathways at an early age may raise attainment and participation for some students at the expense of reduced attainment for others. Rigid barriers can make movement between pathways difficult.**

Germany and Singapore have pathways stratified by attainment from the beginning of secondary education. In Singapore, there are three streams at lower secondary level leading to different examinations (including O levels) at age 16, with some students taking a longer course leading to O levels at age 17. Singapore takes steps to enable movement between the pathways, but there are limited data on how effective these are. Germany separates students into different academic and vocational schools at lower secondary. Mathematics education outside school – shadow education – increases at transition points and is stratified by socio-economic status, which may in turn increase inequities in participation (Bray, 2007).

There is international and national evidence to suggest that upper secondary educational attainment and participation is stratified by gender and socio-economic status (Gorard, See & Davies, 2012; Nash, 2000). Although evidence from PISA (OECD, 2012) indicates that early selection into either academic or vocational pathways is associated with reduced quality and increased inequity, most studies find that inequity does not increase as a result of different pathways at upper secondary (e.g. Sullivan, Heath & Rothon, 2011). In New Zealand and Scotland, the two countries where national data were available on this issue at upper secondary level, there was no evidence of participation stratified by socio-economic status after controlling for prior attainment. Nevertheless, any introduction of an additional advanced mathematics qualification should be carefully monitored for any inequities in participation.

Recommendation 11: Participation in upper secondary mathematics by gender and socio-economic status, and any adverse consequences of new pathways (such as the growth of shadow education and potential stratification by socio-economic status), should be monitored.

6. Vocational education

- Vocational education tends to have a lower status than general education but this is less pronounced in Germany and Singapore, where vocational education is highly valued relative to other countries.
- Vocational qualifications and pathways in England are relatively complex and the number of mathematics components completed is highly variable from year to year.

Vocational education appears to have a lower social and economic status than general education pathways, even in Germany. However, Singapore and Germany appear to have managed to make vocational education an attractive and respected option. In Hong Kong, Massachusetts and New Zealand, vocational education formed only a very small part of upper secondary education.

In England, participation and provision in vocational education is poorly documented and thus poorly understood. This is also the case in most other countries. In England, many post-16 students in vocational education are studying mathematics below the level they have already attained (Royal Academy of Engineering, 2011). In addition, the number of students taking mathematics qualifications fluctuates very significantly from year to year, suggesting that this provision is particularly responsive to institutional factors other than student demand or need (e.g. funding, targets or league tables), which may exacerbate the problem.

Recommendation 12: Mathematics qualifications in vocational education need to be simplified, and incentives should be carefully designed to avoid students taking qualifications that are below the level they have already attained.

Recommendation 13: More reliable and valid data on students' participation in both basic and advanced mathematics in vocational education are needed. These data should include student background variables so that any social stratification can be monitored and addressed.

7. Policy development and implementation

- Some countries appear to have achieved a more coherent and consensual approach to the development and implementation of policy than England.
- One country, Hong Kong, has succeeded in introducing compulsory mathematics into upper secondary education.

The smaller countries in the study appear to be marked by a coherent and consensual approach to policy. Key features appear to be longer timelines for the development and implementation of policy, and inclusive approaches to policy-making. The needs of higher education and employers in Germany, New Zealand and Singapore appear to be more effectively recognised than in England. However, a strong note of caution is necessary here. National policy processes are deeply embedded in socio-cultural and historic practices and cannot be transplanted from one system to another. Indeed, to an outsider, these policy processes may appear smoother than they actually are. In addition, it is unlikely that the policy-making process in England will change radically in the short or medium term.

Hong Kong's introduction of compulsory basic mathematics into upper secondary education has been as part of a wider reform to secondary and tertiary education. As a result, participation in upper secondary mathematics has increased from less than 50% to more than 95%. The transition was relatively straightforward because of high societal expectations for mathematics, a relatively long timescale for implementing the new arrangements, and sufficient well-qualified teachers to meet the increased demands. Introducing near universal participation in mathematics at upper secondary in England is likely to involve a greater systemic upheaval. A key issue that could nonetheless be addressed in the medium term is the capacity of the existing teaching workforce, particularly given the existing shortage of mathematics teachers (Smith, 2004).

Recommendation 14: The success of any new qualification will be dependent on ensuring it is widely available and supported by appropriately designed performance measures. Strong incentives should be provided for schools and colleges to offer *all* advanced mathematics options.

Recommendation 15: It is particularly important to develop a consensual approach to the implementation of the policy and to ensure the support of all stakeholders. This is likely to be facilitated by an extended timescale of development and implementation.

Recommendation 16: Research, including modelling scenarios, is necessary to understand how many more mathematics teachers are needed, the extent to which existing teachers can be retrained, whether former teachers can be attracted back, and how any potential negative effects on lower secondary or existing advanced mathematics routes can be avoided.

3. Methodology

We have summarised our methodology here, but a more detailed account is available in an accompanying technical report published online at www.nuffieldfoundation.org/towards-universal-participation. The work was carried out between April and October 2012 in three stages.

Stage 1: Academic literature search

We drew up a list of search terms and a list of appropriate literature databases based on previously used sources, awareness of the area and recommendations from others (full details of both lists are available in the technical report). We then applied the search term list for each country across each database. We also conducted additional searches, both by hand and online, with a particular focus on areas where the literature appeared more limited. Following a process of exclusion, we were left with 743 publications, which we coded and ranked according to their relevance to our interests. In relation to our interests, the review highlighted substantial gaps in the current literature. For example, there is very little comparative research across countries, and very little research into mathematics options within vocational courses.

Stage 2: Policy publications and production of country profiles

For each of the countries, we compiled a detailed profile using existing sources, such as policy documents, statistical data and curricular documents. Each profile was then validated by experts from each country to ensure it was accurate and complete. We also asked the experts to compare what happens in theory (according to policy) with what happens in practice (according to their experience or knowledge of the education system in which they work).

Stage 3: Review, synthesis and report writing

As a team, we discussed the coded literature and completed country profiles to identify literature and information relevant to our research questions and emerging themes. Individual team members took responsibility for different aspects of the report, which were then brought together for further discussion. Where necessary, aspects of the report were further validated by country experts.

Limitations of comparative research and data

While we believe that the evidence presented here is sufficiently robust for our purposes, it is important to bear in mind that any comparison between educational systems is highly problematic. Educational is deeply embedded in national cultural values and political economies. Statistics between countries are not absolutely comparable. Different countries collect and represent their data in different ways, and we have made calculations and judgements based on the available data which may increase the margin of error. Moreover, the level and extent of mathematics at advanced level varies between countries, although recent reforms in Hong Kong and Singapore have reduced this variation (see also Ofqual, 2012).

For some aspects – particularly those where publicly available data were limited due to confidentiality or the political context of the country – we have relied on the professional knowledge and judgement of our country experts, although we have addressed the possibility of individual bias by using multiple experts for each country.

In addition, much of the evidence that we have collected either through the country profiles or the literature is largely descriptive and does not in itself provide evidence of causation. As a result, our synthesis of the evidence highlights the factors and the features of educational countries that we, and our country experts, judged to be important (or not).

4. The countries surveyed: a broad comparison

Seven countries were included in the study.

- Two high-attaining Pacific Rim countries – Hong Kong and Singapore – were included as there is substantial interest in these in relation to policy developments in England.
- Two 'near-neighbours' – New Zealand and the United States (Massachusetts) – were included due to their high participation and achievement levels respectively, in comparison with UK countries.
- Germany (with a focus on Rhineland-Palatinate)[1] provides a point of comparison within Europe and is of particular interest given Germany's strong vocational education.
- England is included because this study is intended to inform education policy and practice here. Scotland is of interest because it has greater participation in

1 Information from a second Land – Baden-Württemberg – was also obtained but this is only included, where appropriate, for comparative purposes.

basic and advanced mathematics than England, as well as a significantly different educational system.

TABLE 1: A TOP-LEVEL COMPARISON OF UPPER SECONDARY EDUCATION ACROSS THE COUNTRIES INCLUDED IN THIS STUDY.						
	Upper secondary age range?	Number of upper secondary years?	Is upper secondary education compulsory?	Is mathematics compulsory in general education?	Is mathematics compulsory in vocational education?	Are other subjects compulsory?
England	16 – 18	2	No (but will be from 2015)	No	Some	No
Germany (R-P)	15/16 – 18	2 – 3	Yes, at least part-time	Yes	Yes	Yes
Hong Kong	15 – 18	3	No	Yes	N/A	Yes
New Zealand	15 – 18	3	Yes, to age 16 (but this is likely to increase)	To a basic level	N/A	Yes
Scotland	16 – 17/18	1 – 2	No	No	Some	No
Singapore	16 – 18/19	2 – 3	No	Some	Some	Yes
USA (MA)	15 – 18	3	Yes, but only to age 16 in Massachusetts	Yes	N/A	Yes

Country profiles

We have included short profiles highlighting the key features of each country relating to upper secondary mathematics. More in-depth information is available in the full country profiles, which are available to download from www.nuffieldfoundation.org/towards-universal-participation.

ENGLAND

- Education is overseen by the Department for Education (DfE). Historically, state-maintained schools have been the responsibility of local authorities, although recent reforms have enabled a large proportion of schools to become independent academies reporting directly to the DfE.
- Schooling is compulsory from the age of five, although many children begin nursery education at the age of three. Education is currently compulsory until the age of 16, but this will increase to age 17 by 2013 and age 18 by 2015.
- England has a National Curriculum for mathematics, which is currently under review, although this does not cover upper secondary education. Students take national secondary school exit examinations (GCSEs) at the end of compulsory schooling (at age 16).
- After age 16, students may remain in secondary school for a further two years, transfer to further education colleges or other education/work-based training providers, or leave education. For those who remain in education and training, there are a vast number of courses and qualification options available. These include traditional/general options (taken by the majority of students in secondary schools), vocational courses, and routes which cross between the two systems (such as Diplomas and Apprenticeships).
- Traditional routes include AS and A level qualifications, which are narrow specialisations. Mathematics is not compulsory, but students who choose this subject have options including AS or A level in Mathematics, Use of Mathematics and Further Mathematics.
- Although other qualifications such as the International Baccalaureate and Free-Standing Mathematics Qualifications (FSMQs) are available, take-up is very low and there is effectively no mathematics qualification in general education for those that have achieved basic mathematics but who do not choose A level Mathematics.
- Participation rates in any mathematics at upper secondary level are very low in comparison to other countries, and relatively low in advanced mathematics (despite increases in recent years).
- There is a wide range of vocational qualifications, which generally have a lower status than traditional routes. The type and extent of mathematics varies, but for many students, the mathematics in vocational courses will be below the level they have already achieved. Vocational education is currently under review in England.

GERMANY (RHINELAND-PALATINATE)

- Germany is a federal country in which education is devolved to the governments of the federal states (Länder). The country is currently undergoing reform towards a more centralised educational system in response to PISA results, although there are still considerable differences between Länder.
- The age of compulsory education varies across Länder although some upper secondary education is compulsory whether through a general or vocational route. There is no equivalent to GCSE 'exit' examinations at age 16.
- Teachers are trained specifically for different stages in education, including upper secondary mathematics and for general teaching in vocational schools.
- Students are stratified into different secondary routes: Gymnasium (which prepares students for university entrance), Realschule (which provides a broad-ranging education for intermediate achieving students), Hauptschule (which prepares students for vocational education) or Gesamtschule (a comprehensive school). The Realschule and Hauptschule offer only lower secondary education. At upper secondary level, vocational education is provided through the Duales System (which combines apprenticeship with vocational schooling) or in the Berufsfachschule (which provides school-based VET), and technical education in the Fachoberschule.
- At age 16+, students receive either general education or vocational education (approximately 20% and 80% respectively). Students in the Gymnasium must meet achievement standards to progress to upper secondary, where they work towards the Abitur (equivalent to A level) as their final examination. The Abitur allows access to university.
- The content and administration of the Abitur varies across Länder. In Rhineland-Palatinate, questions are set by teachers (subject to Ministry approval).
- Mathematics is compulsory on almost all general and vocational pathways. However, this varies between a basic or advanced course (in general education), and the type of vocational course and whether or not students will take the Abitur and then progress to university (in vocational education).
- A greater proportion of students achieve a basic, or Level 2, qualification in mathematics than in England.

HONG KONG

- Hong Kong is remarkable for its high performance in international assessments of reading, mathematics and science. Students are expected to be trilingual in Cantonese, Putonghua (Mandarin) and English.
- Secondary education has recently shifted away from historic alignment with England, and is now aligned with tertiary education in China and the USA. The duration of secondary education has been reduced by one year and now corresponds with mainland China and the USA, where degree courses are typically four years long.
- Few students leave school before upper secondary education and there is no lower secondary school leaving qualification.
- Upper secondary education incorporates general education and applied subject options in schools; few students of upper secondary age are in vocational education institutions.
- The Hong Kong A level Examination has been phased out and replaced with the Hong Kong Diploma of Secondary Education (HKDSE). There are four core subjects: Chinese Language, English Language, Mathematics and Liberal Studies. Although not formally compulsory, almost all students accept the recent entitlement to upper secondary education and therefore study these core subjects.
- Mathematics is a high status subject. Although the proportion of students taking mathematics in upper secondary has increased very significantly, Hong Kong's school workforce appears to have made the transition from optional to compulsory mathematics with relative ease. Participation in advanced mathematics appears to have reduced slightly.
- There are no unit choices in the basic mathematics course but students can choose to take one of two more advanced mathematics options, focusing on calculus and just one of either statistics or algebra. About 22% choose one of the advanced options.
- Teachers are generally highly trained and there are relatively long timescales for the implementation of education reforms.
- There is extensive use of shadow education throughout schooling.

NEW ZEALAND

- Education is compulsory until age 16. However, current policy initiatives are working towards increased participation in upper secondary. New Zealand has a decentralised system, with each school responsible for its day-to-day running.
- There has been a high level of concern about standards over time, although New Zealand appears to have achieved a relatively consensual approach to the development of educational policy.
- In primary and lower secondary, students are assessed through teacher professional judgement and formative assessment. There is no equivalent to GCSE 'exit' examinations at age 16.
- Each year of senior secondary corresponds approximately with Levels 1–3 of the National Certificate of Education Achievement (NCEA). This is a credit-based assessment system which leads towards university entrance. Degree courses are generally four years long.
- There are no compulsory subjects at senior secondary, although students are required to study numeracy and literacy at a basic level within NCEA Level 1 (usually Year 11). Students generally choose to study a broad range of subjects. Schools also have a high level of flexibility to adapt courses to students' needs.
- Two distinct subjects are available in advanced mathematics: mathematics with calculus, and mathematics with statistics. Students can choose to take a greater or smaller amount of mathematics in each subject and can choose to take modules from both subjects. The modules in mathematics with statistics are smaller, allowing students who would not otherwise choose advanced mathematics more flexibility in their choices. Hence, for many students who study mathematics with statistics, advanced mathematics in upper secondary involves less content than in most comparable countries.
- Participation levels in mathematics are very high and the different subject options appear to be a key feature in contributing to this. Mathematics with statistics is the most popular subject at NCEA Level 3 and is perceived by higher education, employers and students as relevant to a broad range of higher education courses and career options.
- Upper secondary is almost wholly school-based and general (as opposed to vocational).

SCOTLAND

- Education policy is devolved to the Scottish government. The Scottish education system has evolved very differently to the rest of the UK, although England is nevertheless a strong influence.
- A new curriculum – the Curriculum for Excellence – was implemented in 2010 for students at the beginning of secondary education, and the first new National Qualifications will be taken in 2013–2014. It is not expected that upper secondary mathematics will change much under the Curriculum for Excellence as the current programme is popular and relatively successful.
- Education is compulsory until age 16.
- Early entry to national examinations at age 16 has increased in recent years but is expected to decrease under the new system.
- Scotland emphasises breadth of study as well as depth. Lower secondary students cover twelve subjects in S1 and S2, eight subjects in S3 and S4 (culminating in Standard Grade examinations), five subjects in S5 (culminating in Highers), and three subjects in S6 (culminating in Advanced Highers).
- Degree courses in Scottish universities are typically four years long and students may progress to university after S5. Those staying in school to the end of S6 and taking Advanced Highers may start in the second year of some courses.
- Mathematics is not compulsory at upper secondary level, but curriculum breadth leads to relatively high participation. Of those who took courses in state-funded schools, 23% took Higher Mathematics and 5% took Higher and Advanced Higher Mathematics.
- The Higher Mathematics qualification is less extensive and roughly equivalent to the core pure mathematics modules of AS Mathematics in England. Hence, for most students, advanced mathematics in upper secondary involves less content than in most comparable countries.
- Students may opt to take vocational qualifications such as Scottish Vocational Qualifications (SVQs). These are tailored to specific industry requirements and some courses include mathematical content. Vocational education appears to have been strongly influenced by England.

SINGAPORE

- The Singaporean education system is small and centralised, with strong historic links to the English system. GCE O and A level qualifications are still offered, although these have been reformed to meet local needs. Students taking A levels are required to take a contrasting subject. Arts and humanities students must take a science or mathematics option and vice versa.
- Evaluation and reform of the education system is undertaken periodically. There has been considerable investment over the past 30–40 years, and students perform at a high level on PISA and TIMSS.
- There is heavy setting and streaming, and high stakes examinations throughout the country, although in recent years there has been greater recognition of other forms of achievement.
- Teaching is a high status profession and there is significant investment in professional development.
- Students are placed in one of three streams for secondary education: Express (for the top 60% of students working towards GCE O level), Normal Academic (for the next 25% leading towards the GCE Normal/Academic examination with the possibility of taking the GCE O level the following year), and Normal Technical (for the bottom 15% with a technical focus, leading to the GCE Normal/Technical examination).
- Upon completion of secondary education, students enter pre-university education (which is offered by Junior Colleges, Centralised Institutes and IPs, and attracts approximately 28% of students) or vocational education (in polytechnics or the Institute of Technical Education). Post-secondary education is not compulsory, but 93% of students participate. Recent reform to the Junior College strand has led to a locally-based curriculum and examination structure.
- Mathematics is not compulsory for A level students, but its high status as a subject and the requirement for students to take a contrasting subject results in high levels of participation (approximately 80% of A level students take some mathematics).
- In vocational pathways, requirements to follow mathematics courses depend on the course being undertaken.
- There is extensive use of shadow education throughout schooling.

UNITED STATES OF AMERICA (MASSACHUSETTS)

- The structure of the education system is determined at both state and local level. There is an emphasis on meeting and maintaining high academic standards, as well as wider concerns over maintaining technological and economic advantage. Teaching time is predominantly structured around heavily market-driven textbooks and other commercial schemes.
- The age of compulsory education varies across states. In Massachusetts, education is compulsory between the ages of six and 16.
- Students in Massachusetts have a higher average level of attainment than the US average (as measured for Grade 8 students in TIMSS 2007).
- Students who complete High School receive the High School Diploma. In Massachusetts the High School Diploma includes compulsory mathematics. Three years of High School is required for entry to most post-secondary education, with four years (including calculus) required for many STEM subjects.
- Massachusetts and most other states follow the recently introduced Common Core State Standards which give students two mathematics options for the first three years of High School: traditional or integrated. During the fourth year, students complete a variety of courses: statistics, discrete mathematics, or pre-calculus.
- Vocational education varies across states but participation is low (1.4% of 15–19–year-olds are involved) and the mathematics does not transfer into general programmes. Massachusetts is slightly different in offering combined modern career technical studies, with general courses including advanced mathematics through *High Schools That Work*.

5. Participation in mathematics

This report's predecessor, *Is the UK an outlier?* (Hodgen et al., 2010) included an examination of rates of participation in upper secondary mathematics. In this report we revisit these data for two reasons. Firstly, to reflect the most recent policy changes, such as the extensive restructuring of the Hong Kong education system, and secondly because it provides a useful background to discussion of the factors that affect participation.

Table 2 shows the participation rates in the seven countries surveyed. We have updated these rates on the basis of new and revised information received,[2] or as a result of clarifications from government statisticians.[3] An explanation of how these participation rates were calculated and more detailed information on participation in each country is available in the technical report.

TABLE 2: WHAT ARE THE PARTICIPATION RATES IN UPPER SECONDARY MATHEMATICS EDUCATION?

	Studying any mathematics	Studying advanced mathematics
England	20%-26%	13%
Germany (Rhineland-Palatinate)	>90%	8%-14%
Hong Kong	>95%	22–23%
New Zealand	71% (Y12), 44% (Y13)	66% (Y12), 40% (Y13)
Scotland	48% (S5), 21% (S6)	27%
Singapore	66%	39%
USA (Massachusetts)	>84%	>16%

Note: The base for the participation rates is the number of students of upper secondary age who are in education, employment or training.

England: The upper limit of those studying any mathematics includes the 6% who retake GCSE.

New Zealand: Figures for both Y12 and Y13 are given.

Scotland: Figures for S5 and S6 are given for studying any mathematics. For advanced mathematics, an aggregated figure is given that indicates the proportion of student completing advanced mathematics at some point during upper secondary education.

Participation in any mathematics

England's participation rate of 20% is considerably lower than any of the other countries surveyed. Scotland, with a participation rate of 48% of 16–17-year-olds is the second lowest. The most significant change is the participation rate in Hong Kong from below 50% to over 95%, because mathematics has recently become compulsory in upper secondary as part of a wider reform to secondary and tertiary education.[4] Hong Kong, the USA and Germany have the highest levels of participation in at least

2 In general, the information available on vocational education was much poorer than for general upper secondary education. However, we were able to obtain considerably better data about vocational education in Singapore. In Hong Kong, recent reforms have incorporated into general education what was previously classified as vocational and, as a result, the data are now much stronger.

3 In the cases of New Zealand and Scotland, we are indebted to government statisticians who went to considerable lengths to answer our questions and queries.

4 Upper secondary education is not compulsory in Hong Kong, although very few students fail to complete it.

basic mathematics. Mathematics is compulsory in upper secondary in the USA,[5] and near-compulsory in Germany.

Participation in advanced mathematics

New Zealand and Singapore have the highest level of participation in advanced mathematics. Scotland's participation, although lower than New Zealand and Singapore, is considerably higher than in England. A key factor in Scotland is the breadth of study at advanced level, in which students generally choose five or more subjects.

England and Germany have the lowest levels of advanced mathematics participation, although in both cases mathematics is one of the subjects with the highest take-up among students at advanced level. Both have participation levels that are considerably higher than that of Russia, the country with the lowest participation rate in advanced mathematics (Hodgen et al., 2010).

The level of advanced mathematics varies between countries. In general, students studying advanced mathematics in New Zealand and Scotland study less advanced mathematics content than in other countries. There appear to be two significant factors underlying this. First, in both countries, university study is generally four-years to attain an honours degree. Second, students at advanced level choose to study a broad range of subjects, but this is at the expense of depth. In contrast, students in Hong Kong and Singapore study more advanced mathematics content at both basic and advanced levels than in other countries. A key factor in this is likely to be the high levels of attainment and expectation in the lower secondary school. Singapore's strategy of encouraging breadth does not appear to have reduced the level of content in advanced mathematics, although fewer students now choose the very advanced H3, or Further Mathematics option.

Participation in mathematics in vocational education

In Hong Kong, New Zealand and Massachusetts (USA), the numbers in vocational education are very small. Both New Zealand and Massachusetts have introduced new pilot vocational initiatives, while Hong Kong has a significant provision of applied subject options within general education. Students taking these options continue to study general, usually basic, mathematics.

Detailed participation data were not available for the level of mathematics in vocational education for England, Scotland and Germany. We were only able to obtain

5 Upper secondary education is compulsory in some states in the USA, but it is not compulsory in Massachusetts.

data for Singapore because one member of our team has a very good understanding of the country and was able to obtain data directly from vocational institutions. In Germany, we used overall course participation data to estimate participation in advanced mathematics in vocational education. The fact that these data are unavailable for many countries is perhaps an indication of the status of vocational education.

In both Germany and Singapore, a large proportion of upper secondary mathematics education takes place in vocational education. In Germany, almost all of the 80% of students in vocational education take at least some mathematics, In Singapore, not all the students in vocational education study mathematics, but nevertheless 41% of all upper secondary students take mathematics through these routes.

6. Understanding participation: what factors matter?

6.1 The effect of compulsion

England is not unique in its lack of compulsory upper secondary education, but is unusual in not having some compulsory mathematics for students continuing education at this level. Two main reasons are often cited for making mathematics compulsory: social needs and demand for STEM (science, technology, engineering and mathematics). Social needs, regularly rehearsed in policy and sometimes referred to as a utility justification (Huckstep, 2007), are concerned with the economic and social implications of a large proportion of young people not achieving a basic GCSE level mathematics qualification (Croll, 2009). For example, Layard et al. (2002) estimate that the vocational skills gap accounts for around half of the 20% productivity gap between the UK and German economies. This concern has recently been addressed in the Wolf Report (2011), which recommends that these learners should be compelled to continue their study of mathematics until this basic level is attained or until they reach the end of compulsory education. This has also been accepted by the government.

The STEM demand argument centres on concerns that many young people who have been through upper secondary education are poorly prepared for STEM careers (Harris, 2012). This supply issue is a concern not only in England but throughout the developed world (Noyes et al., 2011). It is argued that increasing participation in upper secondary mathematics supports a country's economic goals in an age where STEM plays an increasingly important role. It is not clear to us whether the demand for high-level STEM skills is overstated as some argue. However, it is clear that mathematics is a different and more fundamental subject to other STEM disciplines and that the mathematical needs of the economy in general as well as of particular sectors is poorly understood.

The countries in this study with the highest participation in some mathematics all had compulsion (or for Germany, compulsion for almost all students). However, the countries with the highest level of participation in advanced mathematics (New Zealand and Singapore) do not have compulsion. In some countries compulsion is only until a basic level is attained (such as the requirement to study mathematics to NCEA Level 1 in New Zealand).

Massachusetts (USA) requires many core subjects for the award of the High School Diploma and for entry to higher education. However, this level of compulsion does not appear to have raised standards of attainment in mathematics across the board (Kloosterman, 2010), suggesting that compulsion is not a sufficient strategy. Recent reform in Hong Kong has seen mathematics become compulsory, with curriculum content designed to cover "academic and functional mathematical knowledge that is applicable both in further study and the workplace" (Norris, 2012, p. 18). However, Hong Kong's transition was relatively straightforward because most students were already studying mathematics. Making any compulsory element relevant to all learners is an essential consideration across pathways. This is a particular issue in vocational pathways leading to specific occupations, where mathematics has a tendency to be associated with basic options in which learners see little relevance, and which, as Raffe (2011) has found in Scotland, have little to offer lower attainers.

Making mathematics compulsory in England may have undesirable or unintended consequences. Careful thought needs to be given to who would teach upper secondary mathematics and the consequences of this on other aspects of mathematics education. It may also be unpopular with some students who feel that their other subjects are compromised through compulsory mathematics study (Harris, 2012). Further, in addressing the needs of all learners, it is important to consider the level and content of any compulsory mathematics required of students. The compulsory element in New Zealand (NCEA Level 1) is broadly equivalent to GCSE and it could be argued that compulsion at this level does not solve supply and demand concerns or attract the benefits associated with advanced mathematics qualifications (Dolton & Vignoles, 2000).

In New Zealand, students can choose from a number of mathematics options, including mathematics with statistics, and are able to receive credit for individual units within these subjects. It is possible that this flexibility has a positive impact on participation. By contrast, A level Mathematics in England is highly specialised and unusual in combining pure mathematics and the application of mathematics (Ofqual, 2012).

6.2 The effect of a broader curriculum

England has the narrowest curriculum of the countries in the study, many of which require students to study a greater range of subjects, often in the form of a diploma or baccalaureate qualification, and in some cases covering areas not included within a final examination (Ofqual, 2012). It has been questioned whether a broader curriculum would better prepare students for employment, but Dolton & Vignoles (2002) found that following a broad curriculum does not lead to a wage premium, suggesting this is not the case. However, they did find that having mathematics A level was the only subject to have large, positive returns on future earnings. They go on to argue that a broader curriculum may lead to more students choosing mathematics which may carry subsequent benefits.

Sparkes (2000) looked at students' strategic subject choices in relation to the perceived difficulty of each subject. Mathematics (along with foreign languages) was perceived to be more difficult in terms of grades achieved and hence less likely to be chosen (although individual students who found the subject easier were not more likely to choose the subject). However, where students are balancing subject choices to fulfil a points quota – for instance in accessing higher education – a broader curriculum may encourage students to take a greater risk with subjects that are perceived to be difficult and consequently increase participation in mathematics.

In addition, the more narrow the curriculum, the greater the competition between subjects. It is noteworthy that in Scotland, 88% of students taking biology also take mathematics at advanced (Higher) level (see Scotland country profile), while only 40% of those in England do so (Gill, 2012).

In our view, mathematics should not have a unique status within the curriculum (Huckstep, 2007). In countries where upper secondary mathematics is compulsory, it is never the only compulsory subject. However we believe there are good reasons to consider treating mathematics and English/literacy as special cases, given their fundamental importance both to other curriculum subjects and more generally (Wolf, 2011).

6.3 The effect of attitudes and attainment

Prior attainment

It is clear that prior attainment – GCSE and below – has a crucial impact on participation in advanced mathematics. Tripney et al.'s (2010) review of the literature found that high ability students tended to be more likely than lower ability students to take A level Mathematics, where 'ability' was defined as prior attainment. This was also the finding of Noyes (2009) whose regression analysis found high grades at GCSE (B – A*) to be associated with higher levels of participation. A recent DfE (2011) report reached the same conclusion, finding that achievement at or above the expected level in mathematics prior to Key Stage 5 (A level) resulted in a higher likelihood of participation in A level Mathematics, with high attainers at Key Stage 4 (GCSE) almost seven times more likely than average attainers to participate in A Level Mathematics. However, the impact of prior attainment may be sown well before GCSE:

> "The vast majority of A level maths entrants have a history of high attainment in the subject that stretches back at least as far as the end of primary education." (DfE, 2011, p. 51)

The odds of high attainers at the age of seven participating in A level Mathematics in the upper secondary years are nine times higher than for average or low attaining seven year-olds (DfE, 2011, p. 3). However, while attainment is a crucial factor in whether students choose to study advanced mathematics, the evidence suggests that improving attainment across an educational system is very difficult to achieve (Hodgen et al., 2011).

Enjoyment of mathematics

There is some evidence from international comparisons and time series data that, in England, students' high performance in mathematics may have been at the expense of their enjoyment of it. Although England's TIMSS mathematics score increased between 2003 and 2007, the proportion of students who enjoyed mathematics decreased (Sturman et al., 2008). It should also be noted that participation in A level Mathematics increased over the same period (Matthews & Pepper, 2007). In East Asian countries, there is a more widespread acceptance that successful study means making personal sacrifices and that, if there is any enjoyment, it is derived from success in examinations, which afford status. This view is closer to utility than enjoyment but policy makers have sought to challenge this instrumentalism through different approaches to the curriculum, assessment and pedagogy (Askew et al., 2010). In the UK, there is much evidence suggesting that many students in England dislike mathematics and that this is a crucial factor in their choice not to study it (Brown et al., 2008; Nardi & Steward, 2003). However, Leung (2001) cautioned that pleasurable learning had become equated with easy learning in a number of western countries.

Interest in mathematics

Interest is distinct from enjoyment in that it includes both an emotional component and a values component. Interest in mathematics is therefore rooted in an understanding and evaluation of the nature of mathematical practices. It results in certain dispositions and actions. Gorard et al. (2012) reviewed a recent longitudinal study in the USA which found that secondary students' prior attainment in mathematics significantly predicted their interest in mathematics but not vice versa. However, interest in mathematics appears to be a potentially important factor for participation in non-compulsory advanced mathematics (alongside attainment).

In Germany, Köller et al. (2001) found that while interest had no significant effect on attainment during lower secondary education, it did affect course selection. Thus highly interested students were more likely to choose an advanced mathematics course, resulting in higher upper secondary attainment for these students. Gender differences favoured male students' interest, attainment and participation in advanced mathematics.

In England, with a particular emphasis on interest in terms of value rather than emotion, Matthews and Pepper (2007) found that students' perceptions of the usefulness of mathematics for university and careers was a crucial factor in their choice of A level Mathematics. However, students' perceptions of their own competence was also an important factor, particularly for female students. This was problematic because female students were more likely to underestimate their mathematical competence than male students and therefore less likely to pursue further studies in mathematics. More recently, Mujtaba and Reiss (2012, submitted) found that female students in England who intended to study advanced mathematics had lower perceptions of their mathematical competence than boys did. However, the female students' intention to study mathematics appeared to be more motivated by the expectation of personal material gain, suggesting a shift in the balance of factors influencing their decision. It is possible that this reflects recent STEM information campaigns. In addition, teacher encouragement and perceived extrinsic benefits were important factors for participation in advanced mathematics for both male and female students even after statistically controlling for the effect of prior attainment (Mujtaba et al., in preparation).

Confidence in mathematics

The literature on confidence or perceived competence concerns two overlapping constructs called self-concept (in specific domains such as mathematics) and self-efficacy (in specific tasks such as solving equations). Several studies have found strong associations with attainment, including in mathematics (Bandura, 1997; Marshfmut, 1988; Pampaka et al., 2011). However, the associations substantially weaken when prior attainment and socio-economic status are included in analyses – though differentiating prior attainment from prior affect is fraught with difficulties. Explanations of the associations are predominantly theoretical. What evidence there is for the theorised iterative relationship between perceived competence and actual competence is mixed (Gorard et al., 2012).

Furthermore, the nature and extent of the relationship between perceived and actual competence appears to vary with and between socio-cultural contexts (Klassen, 2004; OECD, 2004). Thus high confidence may precede and motivate attainment and participation in a 'self-enhancement model'. This may be more accurate for countries where mathematical ability is emphasised, such as the USA and England. By contrast, low confidence may be motivational in a self-development model, so that high confidence only follows high attainment and participation. This may be more accurate for East Asian countries, where effort in mathematics is emphasised (Askew et al., 2010; Marsh & Martin, 2011).

The emphasis on either ability or effort is important. Research from the USA suggests that students who see ability as fixed are less likely to persist and solve problems than students who see ability as amenable to effort (Dweck, 1999). A related point is a common view among students in England that there is a predetermined level of mathematics study that they cannot possibly exceed (Brown et al., 2008). These findings have implications for students' commitment to progress and participation in mathematics. The assessment for learning literature endorses the emphasis on effort rather than ability. However, it incorporates evidence that students who develop a realistic view of their strengths and weaknesses can make more progress in their learning (Black & Wiliam, 1998, 2009).

Education and career aspirations

Gorard et al., (2012) reviewed several studies from the UK, USA and New Zealand. Recent longitudinal studies in the UK (Cuthbert & Hatch, 2008) and the USA (Beal & Crockett, 2010; Bui, 2007) had found evidence of associations between parents' and students' aspirations and attainment. However, the effect of attainment on aspirations seemed to be greater than the effect of aspirations on attainment. While these sources therefore suggest that it is more important to focus on attainment than aspirations, such a narrow approach could have negative implications for participation.

In contexts where parents' and students' aspirations were relatively low, some studies in the USA had sought to increase participation and attainment in education by offering financial incentives. Here participation refers to attendance rather than subject choices. Nonetheless, it has implications for attainment and therefore potential subject choices. However, the results of these studies have been more positive for attendance than for attainment. There is some evidence that this is because parents and students know how to make arrangements for attendance but do not know how to improve attainment (Gorard et al., 2012). Clearly, financial incentives need to be targeted at participation in interventions that aim to raise attainment as a basis for future participation.

Information, advice and guidance

As we have already noted, students are more likely to participate in advanced mathematics when encouraged to do so by a teacher, and more so where there are strong relationships between teachers and students. Students who understand the value of mathematics in increasing job prospects, earnings and access to higher education are more likely to participate in advanced mathematics options (Mujtaba et al., in preparation). However, evidence from science education indicates that in both primary and secondary schools in England, there is a tendency to conceive of science as leading to an extremely limited range of careers (Archer et al., 2012; Homer et al., 2011). Students need to be aware of the personal benefits of choosing mathematics in 'keeping options open'. Initiatives, such as Singapore's Education and Career Portal, which provides information to support teachers to guide students in their education and career choices, may be valuable, although little rigorous evidence is available about the efficacy of this or other interventions to support career choices.

7. Can we learn from policy in other countries?

The countries featured in this report approach upper secondary mathematics in ways that reflect their respective educational values. Policies that appear effective are embedded in wider socio-cultural values. As others have highlighted, these fundamental differences in national education values and contexts mean that learning lessons from international comparisons is not straightforward, and we should be careful about 'cherry-picking' policies and approaches from overseas (Askew et al., 2010).

While we did not examine the policy frameworks of each country in depth, we did collect evidence on this from our team of experts and were struck by the ways in which other countries appear to achieve a more coherent approach to the development and implementation of policy than in England. The size of the country appears to be a factor; for example in Singapore, it is possible for all school principals to attend the same, albeit large, meeting. However, evidence from larger countries such as Germany suggests that size is not necessarily an essential factor in coherent policy development and implementation.

Pathways and streaming

Singapore separates students into distinct streams leading to different qualifications based on attainment. About 25% of students are enrolled early in their secondary careers on an extended course that can lead to an O level qualification at age 17 rather than age 16. This reflects the 'ability-driven' approach to education that has been a key feature of the country since the 1970s. Germany separates students into different academic and vocational schools at lower secondary. In both cases, the separation into different pathways creates barriers that hinder students moving between them. Introducing these approaches in England would have far reaching implications, and new evidence from OECD suggests that early tracking into general and vocational pathways is an obstacle to quality and equity and should be deferred to upper secondary level (OECD, 2012).

The success of the mathematics with statistics option in New Zealand is made possible by a number of wider policies. Firstly, the structure of upper secondary pathways enables students to combine the option flexibly within a broad set of subjects as either a partial or a full subject. Secondly, statistics is integrated into the mathematics curriculum throughout primary and secondary school. Thirdly, the statistics option is valued by employers and higher education and appears to be widely understood as distinct to mathematics with calculus. This in turn reflects New Zealand's inclusive and consensual approach to the development of educational policy.

Values

The strength of Germany's vocational education is reflected in the country's educational values that emphasise effort, deduction and logic in contrast to England's focus on ability, induction and intuition (McLean, 1995). This is reflected in the relatively narrow specialisation of students in England and the broader foundations laid for both vocational and higher education by German schools. Similarly, the value accorded to mathematics in Hong Kong and Singapore is rooted in cultural contexts where both education and effort are highly valued (Leung et al., 2006; Usiskin, 2012). In both countries, private tutoring, or shadow education, is a very strong feature of education at all levels and provides an important support to students' participation in advanced mathematics in upper secondary (Bray, 2007).

Planned cycles of reform

Both Hong Kong and Singapore organise educational reform on a cyclical basis allowing time for implementation and for the reform to mature. This has two benefits. First, changes in interrelated policy areas including curriculum, assessment and teacher education can be pursued coherently. In particular, professional development can be built into the cycle of reform and key stakeholders can take time to respond to the reform. For example, textbook publishers can develop and trial textbooks and other materials to meet the requirements of curriculum revision. Second, monitoring and evaluation are built into the cyclical reform process, which then informs refinements to policy and future reforms.

Hong Kong has introduced compulsory mathematics into upper secondary education in recent years. Implementation was relatively straightforward because most students were already studying mathematics as a result of an earlier step towards compulsion. Nevertheless, the timescale for implementing the new arrangements made it possible for teachers to complete further training and plan for the implementation of the new curriculum and qualifications.

Developing close relationships with schools and teachers

In Singapore, the Ministry of Education (MOE) maintains a very close relationship with schools. For example, the MOE operates a system of 'close monitoring' which involves senior MOE managers visiting every school to meet with the principal and teachers. These visits are not school inspections (although the MOE do inspect schools regularly), but are opportunities to discuss issues and raise problems. Such close relationships appear to help to develop a consensual approach to policy across the country as well as being a good way to communicate the value placed on teachers by the MOE.

A consensual and inclusive approach to informing policy

In New Zealand, as in England, there has been a long-standing debate in the media about educational standards and whether they have declined over time (Openshaw & Walshaw, 2010). Yet New Zealand appears to have successfully developed a consensual approach to both policy development and guidance for teachers. In particular, the policy process was informed by a synthesis of research evidence (Anthony & Walshaw, 2007) that aimed not only to inform the development of policy but also to build a professional consensus (Ruthven, 2010). The synthesis placed considerable emphasis on negotiating a shared understanding of effective practice across key stakeholders. In so doing, it addressed educational values as well as 'what works' and acknowledged that the process of reform is as much political as it is rational or scientific. This view is also informed by a need for qualifications to remain relevant in a time of change and, like Germany and Singapore, is supported by an analysis of labour market needs (Heymann, 2003; Machin & Vignoles, 2005; Marcenaro-Gutierrez et al., 2007; Wolf, 2011).

8. Abbreviations and terminology

CTE	Career and Technical Education (USA)
DfE	Department for Education (England)
EQF	European Qualifications Framework
FSMQ	Free-Standing Mathematics Qualification (England)
GCE	General Certificate of Education (England)
GCSE	General Certificate of Secondary Education (England)
HE	Higher education
HKDSE	Hong Kong Diploma of Secondary Education (Hong Kong)
ISCED	International Standard Classification of Education

ITE	Institute of Technical Education (Singapore)
JC	Junior College (Singapore)
MCAS	Massachusetts Comprehensive Assessment System (Massachusetts)
MOE	Ministry of Education (Singapore)
NCEA	National Certificate of Educational Achievement (New Zealand)
PISA	Programme for International Student Assessment
STEM	Science, Technology, Engineering and Mathematics
SVQ	Scottish Vocational Qualifications (Scotland)
TIMSS	Trends in International Mathematics and Science Study
VET	Vocational education and training

Advanced mathematics: Studying mathematics at some level equivalent to GCE Mathematics. In practice, this consists of ISCED Level 3/EQF Level 4 study with a minimum content level judged to be equivalent to GCE AS Mathematics in England.

Basic mathematics: Anything at a lower level than advanced mathematics; this may consist of anything above ISCED Level 1. In curriculum terms, this is usually described as equivalent to GCSE Mathematics in England, but with the addition of 'use of mathematics', 'modelling' or 'applications' that may involve a small amount of ISCED Level 3 study.

Further mathematics: Extended study in advanced mathematics equivalent to at least the pure elements of A2 Mathematics and AS Further Mathematics in England.

General education: Indicates pre-university or, more broadly, pre-tertiary general education as delivered by schools and colleges.

O level: GCE O level was a qualification in England until it was replaced by the GCSE in 1988. O levels were normally taken at age 16 and aimed at the higher attaining 25–30% of the cohort. Many countries with historic links to England, such as Singapore, still have a qualification called O level.

Practicum: Teaching experience in school undertaken as part of initial teacher education.

Vocational education: Vocational education and training (VET) within upper secondary education. This covers education programmes and pathways explicitly linked to particular employment sectors or occupations. In many countries, upper secondary vocational education and training is provided partially or wholly by schools and colleges, as a function of the balance between theoretical and practical learning.

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