Towards universal participation in post-16 mathematics: lessons from high-performing countries

Country profile: United States (Massachusetts)

UNITED STATES BACKGROUND INFORMATION	
Population (June 2012):	313, 650, 302[1]
Population aged 5-19 (2010):	61, 673, 000[3]
Registered school students aged 5-19 (2010):	57, 542, 000[5]
Enrolled "on track" (at or above grade level, 2006):	82.7%[7]
Registered school students in High School (Grades 9-12 inclusive, 2010):	15, 171, 000[9]
Number of schools (2009/10):	98, 817[11]

MASSACHUSETTS BACKGROUND INFORMATION	
Population (2010):	6,587,536[2]
Population aged 5-19 (2010):	1,054,168[4]
Registered school students aged 5-19 (2011-1012):	924,111[6]
Registered school students in High School (Grades 9-12 inclusive, 2011-1012):	287,055[8]
Number of schools (2012): Elementary: Middle: High:	 1171 329 296[10]

The USA is a constitutional republic of states each of which has considerable autonomy. Education structure is determined at both the state and local levels. Free public education is universally available for kindergarten through grade 12. Funding for education in the US comes from federal, state and local levels.

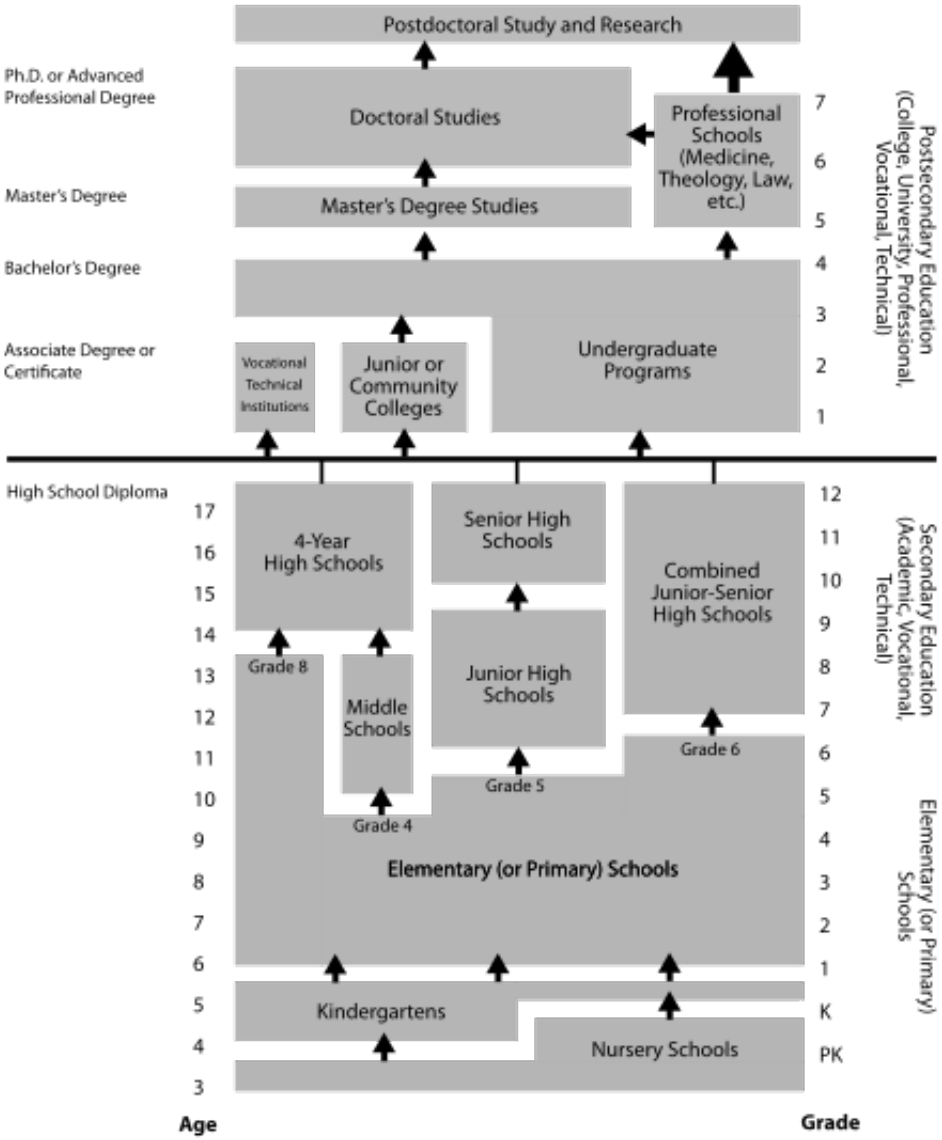
[1] http://www.census.gov/popest/data/national/totals/2011/index.html
[2] http://quickfacts.census.gov/qfd/states/25000.html
[3] http://www.census.gov/hhes/school/data/cps/2010/tables.html
[4] Estimated from statistics at: http://quickfacts.census.gov/qfd/states/25000.html
[5] http://www.census.gov/hhes/school/data/cps/2010/tables.html
[6] Calculated from tables available at: http://www.doe.mass.edu/infoservices/reports/enroll/default.html?yr=1112
[7] See Table 5 in: http://www.census.gov/prod/2009pubs/p70-118.pdf which splits data by student characteristics
[8] Calculated from tables available at: http://www.doe.mass.edu/infoservices/reports/enroll/default.html?yr=1112
[9] Estimated from data table at: http://www.census.gov/hhes/school/data/cps/2010/tab02-01.xls
[10] http://massachusetts.educationbug.org/public-schools/
[11] http://www.infoplease.com/askeds/number-us-public-schools.html

Education is compulsory in all states. The ages for which education is compulsory vary by state, ranging from a starting age of 5-8 to a finishing age of 14-18. In Massachusetts, education is compulsory between the ages of six and sixteen.[12]

Education is divided into three stages:

- Elementary school
- Middle school / Junior High
- High School

Students are divided into Grades based on age. Due to the differing starting and finishing ages, students transfer at different times across the country:



[12] http://www.infoplease.com/ipa/A0112617.html

1. What is the national policy for, and structure of, mathematics education provision for 16-18/19 year-old (pre-university level) learners?

→ Is upper secondary education compulsory or optional?
→ What is the structure of upper secondary programmes?
→ Is any mathematics compulsory in the upper secondary age group?
→ What, if any, are the mathematics options in upper secondary education?

The age at which compulsory education ends varies across states. Education is compulsory in Massachusetts until the age of 16. Therefore, High School is not compulsory.

Across the US, in states that have adopted the new Common Core State Standards (CCSS), within Grades 9-12, students can choose one of two Model Pathways of model courses for the first three years of high school:[13]

- Traditional (Algebra I, Geometry, Algebra II)
- Integrated (Mathematics I, Mathematics II, Mathematics III)

The Massachusetts Department of Elementary and Secondary Education added two additional model advanced courses that students may choose to take after completing either Model Pathway:

- Model Precalculus
- Model Advanced Quantitative Reasoning

Further, Honors, advanced, and Advanced Placement courses are offered in all the core academic subjects in most Massachusetts high schools.[14]

Reflecting Massachusetts' approach, Ohio, which performs above the national average on the National Assessment of Education Progress (NEAP – see Section 5 – Massachusetts scores highest of all states), requires three units of mathematics (including Algebra II) and will require four units from 2014.

In the US generally, there are several options available for a fourth year course that may include Statistics, Discrete Mathematics, or Precalculus as defined by the state or district. Additionally, some students follow an accelerated pathway and are able to complete one of the model pathways in two years instead of three. If that is the case, then students typically complete a Precalculus course and a Calculus course while in high school.

There was a national initiative to write K-12 standards sponsored by the National Governor's Association (NGA) and the Council of Chief State School Officers (CCSSO).[15] 45 states and the District of Columbia have adopted the mathematics standards for implementation over the next couple of years. This year (2011-2012) most states implemented the new standards in grades K-2 fully with transition for 3-12 that focused on the standards that were common to both the state's familiar standards and the Common Core. Next year most will still have a transition, but will include standards that are new to a grade level. Full Implementation is expected in school year 2013-2014. There are also two consortia that are developing

[13] See: http://www.corestandards.org/ - note that some states, for instance Texas, have not adopted the Common Core State Standards and have different graduation pathways
[14] http://www.doe.mass.edu/famcomm/aae.html
[15] http://corestandards.org/

assessments that will be based on the Common Core Standards and will be piloted in the school year 2013-2014 and in place for the 2014-2015 school year.

As a result of the new standards, many states are changing their mathematics requirements for high school graduation. There is a desire to help more students to be more ready for the rigors for college level mathematics, and the hope is that the new standards will assist with that.[16]

The mathematical syllabi and Common Core Standards for Mathematics with guidelines for learning, teaching, and assessment are included within the Massachusetts Curriculum Framework for Mathematics which is divided into pre-K – Grade 8 and High School sections.[17]

MassCore was initiated in Massachusetts in 2007. This is the recommended High School Core Program of Studies for College- and Career-Readiness. It is recommended that students follow:[18]

- four years of English
- four years of Mathematics
- three years of a lab-based Science
- three years of history
- two years of the same foreign language
- one year of an arts program
- five additional core courses
- additional learning opportunities i.e. AP classes, dual enrolment or work-based learning

Massachusetts tries not to have mandated/compulsory programmes; Massachusetts' school districts have some autonomy as a "local control" state. MassCore is a Department of Elementary and Secondary Education recommended program of studies and local school district decision-making is retained. All districts are encouraged to offer courses to fulfil MassCore, but the graduation/diploma requirements are set by the locally elected school committees.

There are incentives for students to complete the MassCore requirements because it is believed that students completing it will increase their college and career readiness. In addition, low income students eligible for the federal Pell Grant who complete MassCore are also eligible to receive an Academic Competitiveness grant from the United States Department of Education.[19]

Also, in support of MassCore, the Board of Higher Education at their March 22, 2011 meeting unanimously adopted a change in their mathematics admissions standards for four-year public higher education institutions effective for the freshman class entering fall 2016. This change requires 4 courses of mathematics (Algebra I & II and Geometry or

[16] Information provided by country expert
[17] Massachusetts Department of Elementary and Secondary Education. (2011). Massachusetts Curriculum Framework for Mathematics, Grades Pre-Kindergarten to 12 Incorporating the Common Core State Standards for Mathematics. Malden, MA: Massachusetts Department of Elementary and Secondary Education.
[18] http://www.doe.mass.edu/ccr/masscore/
[19] http://www2.ed.gov/about/offices/list/ope/ac-smart.html

Trigonometry, or comparable coursework) including mathematics in the final year of high school.[20]

Some schools also offer an International Baccalaureate programme, Early College Programme, or have STEM specific secondary programmes.[21]

2. What are the overall participation rates in mathematics study for 16-18 year-olds both as proportions of students and proportions of the age cohort?

- → What are current levels of participation in mathematics overall amongst the upper secondary cohort and age group?
- → What are the current levels by gender?
- → How have these participation rates changed over time?

The United States

Across the US, the profile of students taking mathematics courses in High School in 2009 was as follows[22].

Mathematics course	Male	Female	Percentage
Any mathematics course	100.0	100.0	100.0
Algebra I	68.5	69.3	68.9
Geometry	86.6	89.9	88.3
Algebra II	73.5	77.6	75.5
Trigonometry	5.8	6.4	6.1
Analysis/pre-calculus	33.8	36.6	35.3
Statistics/probability	10.7	10.9	10.8
Calculus	16.1	15.7	15.9
AP calculus	11.3	10.7	11.0

Changes in the profile of students taking mathematics courses in High Schools across the US are as follows:

Mathematics course	1990	1994	1998	2000	2005	2009
Any mathematics course	99.9	99.8	99.8	99.8	99.9	100.0
Algebra I	63.7	65.8	62.8	61.7	62.8	68.9
Geometry	63.2	70.0	75.1	78.3	83.3	88.3
Algebra II	52.9	61.1	61.7	67.8	70.3	75.5
Trigonometry	9.6	11.7	8.9	7.5	8.3	6.1
Analysis/pre-calculus	13.3	17.3	23.1	26.7	29.5	35.3
Statistics/probability	1.0	2.0	3.7	5.7	7.7	10.8
Calculus	6.5	9.3	11.0	11.6	13.6	15.9
AP calculus	4.1	7.0	6.7	7.9	9.2	11.0

[20] http://www.mass.edu/library/documents/2011-03-22BHEMeetingMinutes.pdf
[21] Information provided by country expert
[22] http://nces.ed.gov/programs/digest/d11/tables/dt11_161.asp

Massachusetts

In 2009 there were 290,502 students enrolled in Grades 9 – 12 in Massachusetts.[23]

In 2009-2010 there were predicted to be 63,650 graduates from High School in Massachusetts.[24]

Participation levels in mathematics and mathematics options vary across districts in Massachusetts.[25] Information on participation in different mathematics options is not collected.

3. What are the patterns of participation in terms of following different routes involving mathematics?

→ What are current levels of participation in different mathematics options amongst the upper secondary cohort and age group?

See section 2.

4. What is the content and level of the different kinds of provision?

→ What is the structure and content of the mathematics options?
→ How is teacher education organised in order to offer the mathematics options?

The United States

Across states which have adopted them, the Standards define what students should understand and be able to do in mathematics.

The content for the Common Core State Standards for High School is divided into six areas:[26]

- Number and Quantity
- Algebra
- Functions
- Modeling (integrated across other areas)
- Geometry
- Statistics and Probability

The following table outlines the content of each area. There are two categories of standards in the 9-12 conceptual categories: College Ready and Beyond College Ready. Standards that are beyond college ready are indicated with a (+) symbol, and indicate standards that would typically be included in a Precalculus course or an Advanced Statistics course. More detailed information is available within the Core Standards document.

[23] http://nces.ed.gov/programs/digest/d11/tables/dt11_036.asp
[24] http://nces.ed.gov/programs/digest/d11/tables/dt11_112.asp
[25] Information provided by country expert
[26] The Core Standards document for mathematics is available at: http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf

Core Standards Area	Mathematical Content	
Number and Quantity	The Real Number System	Quantities
	• Extend the properties of exponents to rational exponents • Use properties of rational and irrational numbers.	• Reason quantitatively and use units to solve problems
	The Complex Number System	Vector and Matrix Quantities
	• Perform arithmetic operations with complex numbers • Represent complex numbers and their operations on the complex plane • Use complex numbers in polynomial identities and equations	• Represent and model with vector quantities. • Perform operations on vectors. • Perform operations on matrices and use matrices in applications.
Algebra	Seeing Structure in Expressions	Arithmetic with Polynomials and Rational Expressions
	• Interpret the structure of expressions • Write expressions in equivalent forms to solve problems	• Perform arithmetic operations on polynomials • Understand the relationship between zeros and factors of polynomials • Use polynomial identities to solve problems • Rewrite rational expressions
	Creating Equations	Reasoning with Equations and Inequalities
	• Create equations that describe numbers or relationships	• Understand solving equations as a process of reasoning and explain the reasoning • Solve equations and inequalities in one variable • Solve systems of equations • Represent and solve equations and inequalities graphically
Functions	Interpreting Functions	Building Functions
	• Understand the concept of a function and use function notation • Interpret functions that arise in applications in terms of the context • Analyze functions using different Representations	• Build a function that models a relationship between two quantities • Build new functions from existing functions

Core Standards Area	Mathematical Content	
	Linear, Quadratic, and Exponential Models	Trigonometric Functions
	• Construct and compare linear, quadratic, and exponential models and solve problems • Interpret expressions for functions in terms of the situation they model	• Extend the domain of trigonometric functions using the unit circle • Model periodic phenomena with trigonometric functions • Prove and apply trigonometric identities
Modeling	Modeling is about linking classroom mathematics and statistics to everyday life. Modeling standards appear throughout the mathematics standards rather than being presented as discrete topics.	
Geometry	Congruence	Similarity, Right Triangles, and Trigonometry
	• Experiment with transformations in the plane • Understand congruence in terms of rigid motions • Prove geometric theorems • Make geometric constructions	• Understand similarity in terms of similarity • Prove theorems involving similarity • Define trigonometric ratios and solve problems involving right triangles • Apply trigonometry to general triangles
	Circles	Expressing Geometric Properties with Equations
	• Understand and apply theorems about circles • Find arc lengths and areas of sectors of circles	• Translate between the geometric description and the equation for a conic section • Use coordinates to prove simple geometric theorems algebraically
	Geometric Measurement and Dimension	Modeling with Geometry
	• Explain volume formulas and use them to solve problems • Visualize relationships between two dimensional and three-dimensional objects	• Apply geometric concepts in modeling situations
Statistics and Probability	Interpreting Categorical and Quantitative Data	Making Inferences and Justifying Conclusions
	• Summarize, represent, and interpret data on a single count or measurement variable • Summarize, represent, and interpret data on two categorical and quantitative variables • Interpret linear models	• Understand and evaluate random processes underlying statistical experiments • Make inferences and justify conclusions from sample surveys, experiments and observational studies

Core Standards Area	Mathematical Content	
	Conditional Probability and the Rules of Probability	Using Probability to Make Decisions
	• Understand independence and conditional probability and use them to interpret data • Use the rules of probability to compute probabilities of compound events in a uniform probability model	• Calculate expected values and use them to solve problems • Use probability to evaluate outcomes of decisions

Massachusetts

Within Massachusetts, the structure and content of the mathematics options are set out in the Massachusetts Curriculum Framework for Mathematics. As stated previously, students may follow the Traditional or Integrated pathway, and having completed either, may take the Advanced Model Courses. The Model Traditional Pathway reflects the approach typically seen in the U.S., consisting of two model algebra courses with some Statistics and Probability standards included, and a model geometry course, with some Number and Quantity standards and some Statistics and Probability standards included. The Model Integrated Pathway reflects the approach typically seen internationally, consisting of a sequence of three model courses, each of which includes Number and Quantity, Algebra, Functions, Geometry, and Statistics and Probability standards. [27]

The Framework provides conceptual categories and Content Standards for each course which runs to several pages each of the document. The critical areas of each course are:

Model Algebra I

- deepen and extend understanding of linear and exponential relationships
- contrast linear and exponential relationships with each other and engage in methods for analyzing, solving, and using quadratic functions
- extend the laws of exponents to square and cube roots
- apply linear models to data that exhibit a linear trend

Model Geometry

- establish criteria for congruence of triangles based on rigid motions
- establish criteria for similarity of triangles based on dilations and proportional reasoning
- informally develop explanations of circumference, area, and volume formulas
- apply the Pythagorean Theorem to the coordinate plan
- prove basic geometric theorems
- extend work with probability

Model Algebra II

[27] Massachusetts Department of Elementary and Secondary Education. (2011). Massachusetts Curriculum Framework for Mathematics, Grades Pre-Kindergarten to 12 Incorporating the Common Core State Standards for Mathematics. Malden, MA: Massachusetts Department of Elementary and Secondary Education.

- relate arithmetic of rational expressions to arithmetic of rational numbers
- expand understandings of functions and graphing to include trigonometric functions
- synthesize and generalize functions and extend understanding of exponential functions to logarithmic functions
- relate data display and summary statistics to probability and explore a variety of data collection methods

Model Mathematics I

- extend understanding of numerical manipulation to algebraic manipulation
- synthesize understanding of function
- deepen and extend understanding of linear relationships
- apply linear models to data that exhibit a linear trend
- establish criteria for congruence based on rigid motions
- apply the Pythagorean Theorem to the coordinate plane

Model Mathematics II

- extend the laws of exponents to rational exponents
- compare key characteristics of quadratic functions with those of linear and exponential functions
- create and solve equations and inequalities involving linear, exponential, and quadratic expressions
- extend work with probability
- establish criteria for similarity of triangles based on dilations and proportional reasoning

Model Mathematics III

- apply methods from probability and statistics to draw inferences and conclusions from data
- expand understanding of functions to include polynomial, rational, and radical functions
- expand right triangle trigonometry to include general triangles
- consolidate functions and geometry to create models and solve contextual problems

Model Precalculus

- extend work with complex numbers
- expand understanding of logarithms and exponential functions
- use characteristics of polynomial and rational functions to sketch graphs of those functions
- perform operations with vectors

Model Advanced Quantitative Reasoning

- critique quantitative data
- investigate and apply various mathematical models
- explore and apply concepts of vectors and matrices to model and solve real-world problems

Teacher Training - The United States

In the US, teacher training is similar across secondary phases. There is a federal standard for "Highly Qualified Teachers." To be deemed highly qualified, teachers must have:

1. a bachelor's degree
2. full state certification or licensure
3. prove that they know each subject they teach. [28]

There is no special training for teaching Upper Secondary level mathematics. Teachers teaching beyond Grade 6 are usually specialist teachers. In the same ways states have control over educational policy, they also have quite a degree of control over teacher training. Teacher training can be provided by universities, community colleges, or institutions of higher education. However, teachers must generally have a baccalaureate degree, and that is not available at a community college. An exception would be an early childhood educator (preschool or nursery school), because it is possible to earn an early childhood associates degree and teach in some preschool programmes. There are four main routes through teacher training:

- Blended or concurrent - programmes for students without a bachelor's degree which combines teacher training with a degree course.
- Pre-internship -courses in subject matter development which aims to help student get places on teacher training courses.
- Internship - Students with a degree and receive supported classroom experience whilst they undertake course work in educational theory and teaching strategies for their teaching credential. Such courses usually take two or three years.
- Post-baccalaureate - consecutive courses where a teacher training course follows the bachelor degree. Typically, courses are two or three semesters long and include a placement in school. Some include a Masters degree.[29]

There is currently concern across the US that many teacher education programs are weak with low entry standards, hence attracting poorly qualified students (noted to be in contrast to high performing countries). Mathematics is noted as a critical shortage area. It is intended to bring in institutional reporting and state accountability to address these concerns.[30]

Teacher training issues specific to Massachusetts

In Massachusetts, trainee teachers may be prepared for licensing in undergraduate and post-baccalaureate programmes. To become a teacher in Massachusetts, new teachers must pass the Massachusetts Tests for Educator Licensure (MTEL). For mathematics teachers, this includes a mathematics subject knowledge test.

In order to improve students' achievement in mathematics, Massachusetts has invested funding in enhancing the content knowledge and teaching skills of classroom teachers through the Massachusetts Mathematics and Science Partnership Program (MMSP). This involved partnerships with Higher Education STEM departments. District leadership has built mathematics and science learning communities that are content focused, meaningful to

[28] http://www2.ed.gov/nclb/methods/teachers/hqtflexibility.html
[29] http://www.inca.org.uk/326.html#7.2.1
[30] United States Department of Education. (2011). Our Future, Our Teachers: The Obama Administration's Plan for Teacher Education Reform and Improvement. Washington, DC: United States Department of Education.

teachers, translate content knowledge to classroom practice, and sometimes include higher education mathematics professors.[31]

All teachers are required to continue their professional development in order to move from an initial licence to a professional licence or to maintain their professional licence. Also, there may be monetary incentives included in their district contracts.

The Department of Elementary and Secondary Education (ESE) requires that teachers earn 150 professional development points (PDPs) every five years to recertify for their licence.[32] The Office of Curriculum and Instruction offers a range of professional development for in-service teachers each summer.[33] Also, the District and School Assistance Centers[34] (DSAC) have professional development opportunities.

Also the Association of Mathematics Teachers in Massachusetts (ATMIM) is an active professional organization related to the National Council of Teachers of Mathematics (NCTM). ATMIM hosts a variety of conferences and presentations to encourage communication between teachers and the exchange of useful classroom information. Participation in ATMIM events is not eligible for PDPs, but presentation at an ATMIM conference could be.

5. How are the different mathematics options assessed?

→ How and when are students assessed for summative purposes?
→ Are any alternative assessment pathways available?

High School Diploma

If students complete their education (Grade 12 – age 18) they receive their High School Diploma. High School Diplomas represent a variety of different curricula and standards. 25 states have exit exams as part of their graduation requirements with their use growing more prevalent over the last decade.[35] In Massachusetts, mathematics is followed by all students who pursue the Massachusetts Comprehensive Assessment System (MCAS) leading to the Massachusetts version of a high school diploma.[36]

Alternative Provision

There are provisions for students with disabilities and English language learners. The MCAS Alternate Assessment[37] is available for students with disabilities.

Other US-Wide Assessments

Across the US there are national tests. Participation is voluntary and the types of test available include Scholastic Aptitude Tests (SATs), American College Testing (ACT) and Advanced Placement (AP) examinations, which all assess students' suitability for admission

[31] http://www.doe.mass.edu/omste/SummaryTitleIIB.html
[32] http://www.doe.mass.edu/recert/2000guidelines/guidelines.pdf
[33] http://www.doe.mass.edu/candi/institutes/12/
[34] http://www.doe.mass.edu/apa/sss/support/
[35] Ujifusa, A. (2012). Exit Exams Face Pinch in Common-Core Push, Education Week, pp. 1-19, Wednesday, October 13, 2012.
[36] http://www.inca.org.uk/usa.html
[37] http://www.doe.mass.edu/mcas/participation/?section=all

to higher education. SATs and ACT are used as college admission tools. Although not all postsecondary institutions require either assessment, most require one or the other.

The Scholastic Achievement Test (SAT):

- The SAT I is primarily a multiple-choice test that measures verbal and mathematical reasoning abilities. The test is divided into seven half-hour sections: three verbal; three mathematical; and one additional section, which is either verbal or mathematical. This last section is intended to ensure the same level of difficulty from year to year and does not count towards a student's final score.

- The SAT II, which measures subject knowledge in a specific subject area, is also primarily multiple choice, but only lasts one hour. There are varied subjects to choose from, mathematics included.

The ACT[38]

The ACT contains five assessments:

1. English
2. Mathematics
3. Reading
4. Science
5. An optional writing section

It is a multiple choice test and is used predominantly as part of the admissions process for postsecondary institutions. The mathematics section includes pre-algebra, algebra, coordinate and plane geometry, and trigonometry.

National Assessment of Educational Progress (NAEP)[39]

The U.S. Department of Education, through the National Center for Education Statistics, is the authority for the NAEP testing and began administering the assessment in 1990. There are NAEP assessments in:

- Mathematics
- Reading
- Science
- Writing
- Civics
- Economics
- U.S. history

The same assessments are used nationally to maintain uniformity across the country. Participation in NAEP is mandatory, but it is not given to all students. It is administered to statistically representative populations of students in grades 4, 8, and 12 in all 50 states and the District of Columbia; students are selected by the National Center for Education Statistics.

[38] http://www.act.org/aap/
[39] http://nces.ed.gov/nationsreportcard/

The mathematics assessment includes multiple-choice and constructed response questions on five areas: number properties and operations; measurement; geometry; data analysis, statistics, and probability; and algebra. Additionally items are classified by mathematical complexity.

Advanced Placement (AP) Program[40]

The Advanced Placement Program includes more than 30 college-level courses in multiple subject areas. AP courses are offered by specially trained teachers in a high school setting. The mathematics courses offered are Calculus AB,[41] Calculus BC, and Statistics.[42] There is a standard examination given in May for each AP course, and students may earn college credit or advanced standing through their performance on the examination.

Assessment Options under Development

The *Partnership for Assessment of Readiness for College and Career* (PARCC)[43] and the *Smarter Balance Assessment Consortium*[44] are developing assessments that will be nationally standardized end of course examinations based on the Common Core State Standards.

6. What information is available on students' learning outcomes in secondary education?

→ What research or policy evidence is available on students' expectations, attitudes and attainment in relation to mathematics in lower and upper secondary education?

The US scored 487 in the mathematics test in PISA 2009. This is statistically significantly below the OECD average.[45]

Massachusetts was a benchmarking participant in TIMSS 2007. Grade 8 students produced an average scale score in mathematics of 547. This compares to the US average scale score of 508. Massachusetts has improved its score substantially since 1999:[46]

	1999 average scale score	2007 average scale score
Massachusetts	513	547
US	502	508
England	496	513

MCAS Tests of Spring 2011 Percent of Students at Each Performance Level in mathematics[47]

[40] http://apcentral.collegeboard.com/apc/Controller.jpf
[41] For Calculus AB and Calculus BC course content see: http://apcentral.collegeboard.com/apc/public/repository/ap-calculus-course-description.pdf
[42] For Statistics course content see: http://apcentral.collegeboard.com/apc/public/repository/ap-statistics-course-description.pdf
[43] http://www.parcconline.org/
[44] http://www.smarterbalanced.org/
[45] http://www.oecd.org/document/61/0,3746,en_32252351_32235731_46567613_1_1_1_1,00.html
[46] Mullis, I. V. S., Martin, M. O., Foy, P., Olson, J. F., Preuschoff, C., Erberber, E., . . . Galia, J. (2008). *TIMSS 2007 International Mathematics Report: Findings from IEA's Trends in International Mathematics and Science Study at the Fourth and Eighth Grades.* Chestnut Hill, MA: TIMSS & PIRLS International Study Center, Boston College.

Grade	Proficient or Higher	Advanced	Proficient	Needs Improvement	Warning/ Failing	Students Included
Grade 8	52	23	29	27	21	71,740
Grade 10	77	48	29	16	7	69,342
All Grades (3-10)	58	24	34	27	15	497,712

2009-10 Advanced Placement Performance Report[48]

Subject	Tests Taken	% Score 1-2	% Score 3-5
Mathematics and Computer Science	11,907	32.0	68.0
Calculus AB	5,965	37.4	62.6
Calculus BC	1,668	12.9	87.1
Statistics	3,740	32.5	67.5

AP Scores:
5 Extremely well qualified to receive college credit or advanced placement
4 Well qualified to receive college credit or advanced placement
3 Qualified to receive college credit or advanced placement
2 Possibly qualified to receive college credit or advanced placement
1 No recommendation to receive college credit or advanced placement[49]

2011 Mathematics SAT Performance[50]

Student Group	Massachusetts			US Totals		
	Test Takers	Mean	SD	Test Takers	Mean	SD
Male	29,397	544	116	770,605	531	119
Female	32,416	512	112	876,518	500	113
Total	61,813	527	115	1,647,123	514	117

2011 Mathematics SAT Score Distributions:

Mathematics SAT Score Range	Massachusetts				US Totals			
	Male	Female	Total		Male	Female	Total	
			Number	%			Number	%
700-800	2,913	1,713	4,626	7.5	69215	42678	111893	6.8
600-690	7,262	6,005	13,267	21.5	166746	137291	304037	18.5
500-590	9081	9905	18986	30.7	229454	251716	481170	29.2
400-490	7225	9981	17206	27.8	207920	291024	498944	30.3
300-390	2435	4040	6475	10.5	81011	129634	210645	12.8
200-290	481	772	1253	2.0	16259	24175	40434	2.5

[47] http://profiles.doe.mass.edu/mcas/performance_level.aspx?linkid=32&orgcode=00000000&orgtypecode=0&
[48] http://profiles.doe.mass.edu/adv_placement/ap_perf_dist.aspx?orgcode=00000000&orgtypecode=0&
[49] http://www.collegeboard.com/student/testing/ap/exgrd_set.html
[50] Data extracted from: The College Board. (2011). 2011 College-Bound Seniors Total Group Profile Report Total Group. New York: The College Board and The College Board. (2011). 2011 College-Bound Seniors State Profile Report Massachusetts. New York: The College Board.

Levels of educational attainment of adults (18 years and over) across the US, 2011 (thousands):[51]

	Male	Female	Total
Total	112,301	118,893	231,194
No Education	425	489	914
1st - 4th grade	881	912	1,793
5th - 6th grade	1,855	1,732	3,587
7th - 8th grade	2,303	2,251	4,554
9th grade	1,913	1,939	3,852
10th grade	2,550	2,476	5,026
11th grade	5,839	5,287	11,126
High school graduate	35,082	35,234	70,316
Some college no degree	21,393	23,852	45,245
Associate's degree, occupational	4,169	5,058	9,227
Associate's degree, academic	4,736	6,648	11,384
Bachelor's degree	20,137	21,806	41,943
Master's degree	7,231	8,923	16,154
Professional degree	1,870	1,123	2,993
Doctoral degree	1,916	1,163	3,079

National Assessment of Educational Progress (NAEP)

The most recent NAEP Mathematics Assessment was in 2011 and results were recently released.[52] Overall, the average mathematics score in 2011 is higher than in the eight previous NAEP assessments in both grades. Students in grade 4 in 2011 scored 28 points higher than students in grade 4 in 1990; students in grade 8 in 2011 scored 21 points higher than students in grade 8 in 1990.

The 2009 grade 12 mathematics assessment indicated an increase over the 2005 assessment. For the first time there is not only national results for grade 12, but also 11 states agreed to pilot individual state results for this grade level.[53]

Massachusetts is the highest scoring state in the National Assessment of Educational Progress (NAEP).[54] Indiana also performs above the national average on the NAEP. They have the "Core 40" curriculum program that is similar to MassCore and are also are increasing their mathematics requirement.

[51] Extracted from data tables at: http://www.census.gov/hhes/school/data/cps/2010/tab01-01.xls
[52] http://nationsreportcard.gov/math_2011/
[53] http://nationsreportcard.gov/math_2009/
[54] http://nces.ed.gov/nationsreportcard/

7. What vocational education options are available at upper secondary level?

→ What is the structure and content of the vocational courses available?
→ What status do vocational courses have in comparison to other options?
→ What are the participation levels in these courses?
→ How much mathematics is included in vocational education courses and at what levels?

The United States

Vocational education varies across states in the US, although the emphasis is usually on academic subjects as a result of standards based educational reforms. Vocational education is covered by the Carl D. Perkins Career and Technical Education Improvement Act of 2006. Under this act, vocational education is referred to as "Career and Technical Education".

Employment status and enrollment in vocational courses for the population aged 15-19 and for all adults (thousands, 2010):[55]

	Total			Employed full time			Employed part time			Not employed		
	Total	In vocational courses		Total	In vocational courses		Total	In vocational courses		Total	In vocational courses	
		N	%		N	%		N	%		N	%
Male 15 - 19	10,596	173	1.6	685	23	3.3	1,602	60	3.7	8,309	91	1.1
Male all adults	117,603	1,859	1.6	64,285	997	1.6	10,043	223	2.2	43,275	639	1.5
Female 15-19	10,206	113	1.1	450	10	2.2	1,964	30	1.5	7,792	73	0.9
Female all adults	124,910	2,064	1.7	48,413	893	1.8	17,844	453	2.5	58,653	718	1.2
Total 15-19	20,802	287	1.4	1,135	33	2.9	3,566	90	2.5	16,101	164	1.0
Total all adults	242,513	3,923	1.6	112,698	1,890	1.7	27,887	676	2.4	101,928	1,357	1.3

The Association for Career and Technical Education (ACTE) covers vocational education in the US.[56] They have a variety of information available to different interested parties, including a resource center for educators. This includes lesson plans and resources for professional development.

There is a National Research Centre for Career and Technical Education (NRCCTE).[57] One of their research priorities has been to look at the mathematics taught in Career and Technical Education. This project has produced a curriculum integration model through

[55] Extracted from data table at: http://www.census.gov/hhes/school/data/cps/2010/tab06_10.xls
[56] https://www.acteonline.org/
[57] http://136.165.122.102/mambo/mos/Frontpage/Itemid,1/

mathematics and career and technical education (CTE) teachers working together to enhance mathematics that is embedded in CTE content.[58]

Generally vocational and technical postsecondary degrees and/or certificates are earned through the community college system. A typical career/vocational degree is a two year associates degree, and the certificates may be earned in one, two or three semesters. There may be an assessment of a student's mathematics skills and competencies prior to admittance in a career/vocational programme, with remedial or developmental courses recommended or required as a result of that assessment. Many programmes have developed mathematics courses specifically for the programme, rather than require a general education mathematics course. Typically the mathematics courses designed for career/vocational programmes do not transfer into academic programmes.

Massachusetts

High Schools That Work (HSTW) is a programme designed to combine challenging academic courses and modern career technical studies to raise the achievement of high school students. There are currently 10 educational sites in Massachusetts offering this program. The program aims to combine traditional college preparation with vocational education to improve the mathematics attainment of career-bound students to the national student average. The recommended curriculum includes four years of mathematics including Algebra I&II, Geometry and one higher level course. Various forms of assessment are used in vocational and technical education including a mathematics achievement test.[59]

There are separate curriculum frameworks for VTE programmes, but students are expected to have the same academic courses in addition to their career/technical courses. In addition they are expected to take the same MCAS test and have the same score requirements for VTE students as for all other students to graduate with a diploma. It is possible for VTE student to complete MassCore. Some of the vocational/technical curricula frameworks include additional mathematics, often related to finance or consumer mathematics.[60]

Students in Massachusetts enrol on a vast selection of vocational courses.[61] For the 2011-2012 academic year, there were 57696 students in Massachusetts in Grades 9-12 enrolled in vocational education (32055 male and 25641 female). The table below sets out the numbers involved in each grade for 2011-2012:

Grade	Number of students enrolled in CVTE
9	14817
10	14324
11	14422
12	14072
SP	61
Total	57696

[58] See reports: Capitalizing on Context: Curriculum Integration in Career and Technical Education - http://136.165.122.102/UserFiles/File/Tech_Reports/NRCCTE_Curriculum_WEB_READY.pdf and Stone, J. R. III, Alfeld, C. & Pearson, D. (2008). *Rigor and relevance: Testing a model of enhanced math learning in career and technical education.* American Education Research Journal, 45(3), 767-795.
[59] http://www.doe.mass.edu/cte/resources/hstw/
[60] Extensive information on VET in MA is available from: http://www.doe.mass.edu/cte/
[61] Enrolment data for each course is produced within the tables available from: http://www.doe.mass.edu/infoservices/reports/enroll/cvte12/sg2011_2012.xls and http://www.doe.mass.edu/infoservices/reports/enroll/cvte12/sgr2011_2012.xls

Vocational education options appear to be increasing in status recently in Massachusetts. Several vocational and technical high schools now have waiting lists.[62]

8. What drives the pattern of take-up? How is it linked to the needs of HE, employers and national policy objectives?

- → What are the official criteria, if any, for acceptance to the mathematics options?
- → Are there any unofficial / local criteria for acceptance to the mathematics options?
- → What information, advice or guidance is there about the mathematics options?
- → Are any mathematics recruitment policies targeted to specific groups or types of students?
- → Which subjects and options are students expected or required students to take?
- → Please note the views of, for example, further/higher education institutions, employers, parents or the public more generally
- → Are there 'unofficial' expectations to have particular mathematics qualifications for entry to particular HE courses?

The United States

Academic admission requirements for public and private Higher Education vary greatly across institutions. Admission is often based on students' High School assessments including: grades earned in high school, Grade Point Average, class ranking and SAT or ACT scores. In addition, there may be placement assessments administered to students after they are accepted to a four-year college or university.

Typically a student applying to a four-year postsecondary institution for a liberal arts or social science programme is expected to have at least three years of mathematics, which may or may not include Algebra II. A student applying to a four-year postsecondary institution for a science, technology, engineering, mathematics or in some business majors will be expected to have at least four years of mathematics, and some of those majors may require a calculus course. There is a current trend in many postsecondary institutions to specify that students have a quantitative course in their last year of high school, but the content of a quantitative course is not universally defined.

There are also two-year public intuitions, and most of their programmes do not have admissions requirements, but may have placement requirements for specific courses. To major in postsecondary mathematics there is a higher expectation for secondary mathematics. Generally students are expected to have competed Advanced Placement Calculus (or equivalent) before acceptance to a postsecondary mathematics programme. That may also be true for admission to engineering programmes or to major in physics.

Many states have recognized that students need early guidance in order to make informed decisions about college entrance and have instituted educational planning as part of a student's educational experience. These plans include setting high school graduation and beyond goals that include course selection and are reviewed annually.

[62] Information provided by country expert

Massachusetts

The Board of Higher Education for the public postsecondary institutions has recently amended their mathematics admissions requirements for their four-year institutions for the entering class of 2016.

Massachusetts is piloting Science, Technology, Engineering and Mathematics (STEM) Early College High Schools[63] using federal Race to the Top funding. Students may apply to attend these schools and if there is a large demand it is possible that students will be entered in a lottery for admission. Students may be self-identified, encouraged by teachers or advisors, or encouraged by parents to attend these schools.

Also Boston University (a private postsecondary institution) offers Program in Mathematics for Young Scientists (PROMYS), a six week summer programme for mathematically motivated students to explore mathematical topics. There is also a PROMYS programme for teachers and many Massachusetts secondary high school teachers have attended this programme, then encourage their students to do so as well.

Massachusetts collects data on the intended plans of High School (including vocational options) graduates. This provides useful information on pathways students take / are available to students:[64]

	Number of Graduates	Percentage of Graduates by Plans (2010)								
		Public College		Private College		Other Post-Secondary	Military	Work	Other	Data Not Available
		2-Year	4-Year	2-Year	4-Year					
Total	64,462	21.2	27.5	1.4	31.8	2.5	2.0	7.1	0.7	5.9
Gender										
Male	31,956	21.0	26.2	1.4	27.3	3.1	3.4	9.9	0.8	7.0
Female	32,506	21.4	28.8	1.4	36.2	1.9	0.6	4.3	0.6	4.8
School Type										
Voc/Tech HS	5,943	29.8	18.4	2.1	9.9	5.2	3.8	24.5	0.7	5.5
Charter Schools	919	18.5	30.5	1.5	40.4	1.6	0.9	2.8	0.7	3.2

9. What policies and practices are there for transition and retention?

- → What policies or practices are there to support students' transition from lower secondary to upper secondary mathematics options?
- → More generally, what policies or practices are directed at students struggling with upper secondary mathematics?

The United States

There is concern about students' transition from Middle / Junior High School to High School with research from the National High School Center suggesting that more students fail ninth

[63] http://www.doe.mass.edu/ccr/stem.html
[64] Extracted from data available at: http://www.doe.mass.edu/infoservices/reports/hsg/data.html?yr=10

grade than any other grade. Their research noted that "many students are held back in ninth grade—creating what is known as the ninth grade bulge—and drop out by tenth grade—contributing to the tenth grade dip."[65] This is supported by participation statistics for each grade across the US for 2004-2005:

Grade	8th	9th	10th	11th	12th	Total
Number	3,824,670	4,281,345	3,750,491	3,369,339	3,094,349	18,320,194
Percentage	20.9%	23.4%	20.5%	18.4%	16.9%	100.0%

The authors also note that the ninth grade bulge has almost tripled in the last 30 years. Drawing on an earlier report by Cooney & Bottoms (2002)[66] they identify nine factors that may support students' readiness for high school:

1. Require districts and schools to report annually the percentage of students completing algebra and Freshman English by the end of freshman year
2. Track whether schools are offering more rigorous courses to more ninth-grade students each year
3. Communicate to families what ninth graders are expected to know and be able to do to succeed in high school
4. Require one-on-one planning sessions for all students and their parents for the purpose of planning a rigorous high school program
5. Require high schools to inform middle-grades feeder schools of the percentage of students who completed two years of college-prep English, mathematics and science by the end of 10th grade
6. Provide guidelines on how middle and high schools can work together to prepare students for high school
7. Require and fund high schools to identify eighth graders who are not ready to take college-prep English and mathematics in grade nine and provide a rich summer school experience
8. Provide guidance on how to offer double-doses of catch-up courses—courses that are designed to help students meet the demands of more rigorous high school work, specifically algebra and English, when necessary—in the first semester of high school and enroll them in high school work by the second semester
9. Require districts to report on the outcomes of their transition programs

[65] Kennelly, L., & Monrad, M. (2007). Easing the transition to high school: Research and best practices designed to support high school learning. Washington, DC: National High School Center at the American Institutes for Research. Available from: http://www.betterhighschools.org/pubs/documents/NHSC_TowardEnsuring_051607.pdf

[66] Cooney, S., & Bottoms, G. (2002). Middle Grades to High School: Mending a Weak Link: Research Brief. Atlanta, GA: Southern Regional Education Board. Available from: http://www.eric.ed.gov/ERICWebPortal/contentdelivery/servlet/ERICServlet?accno=ED479785

The original report by Cooney & Bottoms also highlights twelve skills they suggest students should have in order to be ready for College-preparatory Mathematics courses:

1. Read, write, order and represent in a variety of forms: integers, fractions, decimals, percents and numbers written in scientific and exponential notation
2. Add, subtract, multiply and divide integers, fractions, decimals, percents and numbers written in scientific and exponential notation with and without the aid of technology
3. Determine greatest common factors, least common multiples and prime factorization
4. Use ratios and proportions to describe situations and solve multi-step problems
5. Draw and describe different types of angles, polygons and lines using their defining properties and appropriate tools
6. Measure and calculate length, area, perimeter, surface area and volume using appropriate units, tools, techniques, formulas and levels of accuracy
7. Understand and apply the Pythagorean relationship to solve problems
8. Gather, present and interpret data
9. Determine the number of ways an event can occur and the associated probabilities
10. Simplify algebraic expressions and solve equations using substitution, the correct order of operations, grouping and properties of operations
11. Represent, analyze, generalize and extend a variety of patterns with tables, words and symbolic rules
12. Understand the concept of functions and represent them algebraically and graphically

Other approaches to supporting struggling High School students are discussed in the literature. For instance, Duffy (2007)[67] advocates a Response to Intervention (RTI) tiered approach. However, it is not clear how widely spread the use of such intervention programs is in supporting students, how much is aimed specifically at mathematics students, and whether this is successful in reducing drop-out rates.

Dropout prevention and student retention are two areas of concern that were identified in the federal 2010 *Race to the Top* grant proposal.[68] This four year fund was designed to reward states that have demonstrated educational success with the expectation that those states will be able to design programmes that will become best practices for others to follow. Awardees were required to have adopted the Common Core State Standards in order to be eligible to apply.

Massachusetts

From the 2011 data, graduation rates of the four-year cohort in Massachusetts have increased for the fifth consecutive year to 83.4%. The drop-out rate was 7.2%, the lowest since this data was first collected in 2006.[69]

[67] Duffy, H. (2007). Meeting the needs of significantly struggling learners in high school: A look at approaches to tiered intervention. Washington, DC: National High School Center at the American Institutes for Research. Available at: http://www.betterhighschools.org/docs/NHSC_RTIBrief_08-02-07.pdf
[68] http://www2.ed.gov/programs/racetothetop/index.html
[69] http://www.doe.mass.edu/infoservices/reports/gradrates/

		Graduates		Non-Graduates				
	2011 Cohort	4-Year Rate	Difference from 2010	Still in School	Non-Grad Completer	GED	Dropped Out	Expelled
All Students	74,307	83.4%	+1.3	6.6%	1.0%	1.7%	7.2%	0.1%
Female	36,325	86.5%	+1.4	5.0%	1.0%	1.5%	6.0%	0.0%
Male	37,982	80.5%	+1.3	8.1%	1.0%	1.9%	8.3%	0.1%

Support for students varies by district; there is no state-wide policy or practice. However, the ESE Office for Mathematics, Science and Technology/Engineering has several instructional resources available on their website.[70]

Districts are required to put an Educational Proficiency Plan (EPP) in place for students who do not attain a score of 240 on the grade ten mathematics MCAS. The EPP is one of the few policies that are required by state law.[71] The mathematics EPP has three parts: (1) a review of the student's mathematics academic strengths and weakness, (2) additional mathematics course requirements for grades 11 and 12, (3) identification of annual assessments that will be used to determine that the student is making progress toward mathematics proficiency.

There are alternative pathways for students who struggle with academics as well. More information about these programs can be found on the ESE page.[72]

At the postsecondary institutions there are placement tests for mathematics courses. At the Massachusetts public postsecondary institutions (both 2-year and 4-year) the placement test used is Accuplacer.[73] It is mandated by the Board of Higher Education that the Elementary Algebra Accuplacer be used, but in most of the two-year schools and some of the four-year schools it is supplemented by other sections of the Accuplacer or by in-house created placement assessments. There are remedial/developmental mathematics courses available for students who do not place into college credit bearing courses. Some of the remedial/developmental courses include tutoring support and others do not. The support and design of these courses vary at the campuses. The requirement at the four-year institutions, set by the Board of Higher Education, is that no more than 5% of their freshman class be enrolled in remedial/developmental courses. Some students admitted to a four-year institution will be referred to a community college for their remedial courses before being allowed to register at the four-year school.

[70] http://www.doe.mass.edu/omste/
[71] 30.03 Standards for the Competency Determination
[72] http://www.doe.mass.edu/as/pathways/
[73] https://www.accuplacer.org/cat/

10. What information is available on (other) factors affecting recruitment and retention?

→ What factors would you attribute to the upper secondary mathematics recruitment levels in your education system?
→ Please give details of any supporting information or sources

A key factor in educational improvement in Massachusetts over the past decade is political will.[74] Governor Deval Patrick has been a strong supporter of education in Massachusetts. He reinstated the Office of the Secretary of Education as a Cabinet position, and he has several education initiatives including the creation of the Governor's Science, Technology, Engineering and Mathematics Council.[75]

Governor Patrick has actively recruited science based companies to come to Massachusetts and has touted to them that the Massachusetts workforce development through the schools are making every effort to increase the STEM knowledge of the secondary and postsecondary graduates.

[74] Information provided by country expert.
[75] http://www.mass.gov/governor/administration/ltgov/lgcommittee/stem/about/